The Long-Run Redistributive Effects of Monetary Policy*

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Abstract

Using a general equilibrium search-theoretic model of money, I study the long-run distributional effects of monetary policy. In my model, heterogeneous agents trade bilaterally in a frictional market and save using cash and illiquid short-term nominal government bonds. Wealth effects generate slow adjustments in agents' portfolios following their trading activity in decentralized markets, giving rise to a persistent and non-degenerate distribution of assets. The model reproduces the distribution of asset levels and portfolios across households observed in the data. I show that, as wealth inequality increases the incidence of inefficiencies in decentralized trading, policies that improve the ability to self-insure against idiosyncratic shocks are welfare-improving and redistribute resources towards agents that are relatively poor and more liquidity constrained.

Keywords: monetary economics, search theory, heterogeneous agents, public liquidity.
JEL Codes: E21, E32, E52.

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1 Introduction

Central banks commonly use changes in the provision of public liquidity as a way of implementing monetary policy. By using open market operations (OMOs), central banks exchange cash for short-term government bonds. However, there is little understanding of how these interventions can disproportionally affect households with different levels of wealth and portfolio compositions. This paper studies the long-run effects of monetary policy implemented via permanent changes in public liquidity provision while accounting for the observed persistent differences in asset levels and portfolios across households. Further, it examines the distributional consequences of such an implementation and their interaction with the transmission mechanism of monetary policy itself.

Rather than assuming that the economy adjusts instantly and without any cost to a certain targeted interest rate, I explicitly model the transmission mechanism from permanent changes in public liquidity provision to prices and interest rates, and then to the rest of the economic activity. Given that not all agents are receiving the same extra liquidity after an OMO, I conduct the analysis in a context in which changes in the state of the economy (persistent distribution of assets) are fundamental in determining the transmission of monetary policy because of the distributional consequences of changes in public liquidity.

I build a search-theoretic model of money where agents trade sequentially in a frictionless centralized market (CM) and in a frictional decentralized market (DM) that generates the necessity of using money for transactional purposes. In the DM, agents are randomly matched in bilateral meetings. Agents do not have access to credit but, besides holding money, can save in illiquid nominal short-term government bonds. These bonds are illiquid, in the sense that they cannot be used in decentralized trading. However, there is a positive demand for bonds because agents can use them to partially self-insure against idiosyncratic shocks and, therefore, smooth consumption in the CM. A unified fiscal and monetary authority controls the supply of money and nominal bonds. By managing the size and composition of public liquidity in the long run, the monetary authority implements long-run changes in monetary policy and affects the real economy. Such changes in the relative supply of money and government bonds are implemented by means of one-time permanent OMOs.[1]

[1]In practice, the central bank conducts an open market operation by exchanging its liabilities (currency) for other agents' liabilities (e.g., government bonds). The mapping between model and practice is not immediate with a unified fiscal and monetary authority. However, when the central bank conducts an open market purchase of bonds, on top of choosing the supply of money, it also determines the supply

The extent to which agents are disproportionally affected by changes in monetary policy depends on the size and composition of their portfolios. For example, after an OMO in which the central bank buys bonds by injecting more money, agents with large levels of bond holdings are the ones receiving most of that additional liquidity. This type of intervention may have significantly different implications than the monetary injections via lump-sum transfers that have been traditionally studied in the literature and that give every agent the same extra liquidity independent of their location in the distribution of wealth.

Distributional effects are crucial to study the implications of monetary policy in the long run. In the absence of changes in agents' budget sets, OMOs become irrelevant and do not affect the real economy. As discussed by Wallace (1981) and Lucas (1984), if government taxes or transfers offset the distributional changes associated with OMOs, budget sets remain identical across the distribution of agents, and therefore, monetary policy becomes neutral. An environment with frictionless asset markets and with a representative agent would exhibit this particular type of Ricardian equivalence. By endogenously modeling the heterogeneity in asset holdings using an incomplete market setup, my model breaks this irrelevance result and allows us to investigate the consequences of changes in monetary policy in a richer and more realistic setting.

The presence of an interest-bearing asset implies that the insurance role of money is attenuated in my model. Rather than holding large stocks of money, agents are more prone to reduce their usage of money, leaving it mainly for transactional purposes while relying on bonds as their main saving instrument. Of course, the extent to which agents show this behavior depends on the trade-off between the insurance value of money for DM transactions and the insurance role of bonds in the CM. The strength of this trade-off determines portfolios and depends on wealth. Hence, capturing the differences in portfolios along the distribution of wealth observed in the data is critical for determining how long run changes in monetary policy affect different agents. For this reason, I calibrate the model to reproduce the size and composition of households' portfolios in the Survey of Consumer Finances (SCF).

I find that a higher concentration of wealth leads to more inefficient decentralized trading outcomes and reduces total output. Given that money is the only asset used as a means of payment in bilateral meetings, poorer agents are characterized by having

of government bonds available to the public. In this context, OMOs in the model should be interpreted as operations in which the fiscal authority passively and fully accommodates changes in monetary policy. Furthermore, instead of considering OMOs as temporary changes in the supply of government bonds, the model focuses on studying the long-run effects of permanent changes in monetary policy.

portfolios with more money and less illiquid assets. This model result is consistent with the data. However, despite money being an essential asset, it is the total wealth of both agents that determines the opportunity cost of sellers and the buyers' willingness to pay. I show that the more uneven the levels of wealth between buyers and sellers, the higher the wedge between buyers' marginal utility and sellers' marginal cost. Therefore, the more skewed the distribution of wealth is, the more prevalent the inefficiencies in bilateral meetings.

The long-run effects of changes in monetary policy also hinge on how they interact with the distribution of wealth. In contrast to lump-sum transfers, since OMOs swap bonds for money, agents with more bond holdings experience most of the direct effects associated with this policy. The effects of OMOs then depend on how those changes in liquidity spread out from the top of the wealth distribution to the rest of the economy. In this context, for my baseline calibration, I first show that targeting a permanent increase in the nominal interest rate of 1 percentage point (p.p.) requires conducting a one-time permanent OMO that increases the relative supply of government bonds held by the public by 23.6 percent. When this happens, it becomes easier for agents to self-insure against possible expenditure shocks when trading in the DM. The increased insurance opportunities reduce wealth inequality, and hence, boost total output by 0.5 percent following an attenuation of the inefficiencies present in decentralized trading.

Second, I quantify the long-run effects of permanent changes in monetary policy on agents with different levels of wealth. When an OMO increases the relative supply of bonds, the poorest agents gain the most as they increase their total consumption, work less, and become wealthier. This increased level of wealth in the left end of the distribution allows poorer agents to obtain better terms of trade in bilateral meetings, increasing not only their individual welfare but also total output.

I show that policies with lower levels of trend inflation improve welfare because nominal assets lose their value more slowly for lower inflation levels. This effect is even more notorious for the case of trend deflation in the Friedman rule. As a result, wealth is distributed more evenly, and agents trade at lower and less dispersed prices. However, despite this being the case, the output- and welfare-maximizing policy does not eliminate the inefficiencies associated with decentralized trading. This happens as agents trade at a suboptimal level as long as there are differences in wealth between buyers and sellers. The continuous presence of wealth effects and uninsurable risk guarantees that this is the case even for the output-maximizing policy. By the same token, the welfare cost of inflation rises alongside the dispersion in agents' wealth. I estimate that the welfare cost of a 10 percent inflation rate, relative to the baseline of no inflation, is 1.14 percent in terms of consumption.

Related Literature

This paper is related to several strands of the literature. First, the model economy builds on a vast literature of search-theoretic models of monetary exchange that includes papers such as Kiyotaki and Wright (1989), Trejos and Wright (1995), Shi (1997), Lagos and Wright (2005) and Rocheteau and Wright (2005).[2] These papers do not study OMOs but the equilibrium properties of economies where money is essential and where there is no heterogeneity. Their critical reason for trying to work with a degenerate distribution of money is to solve the agents' problem in a manageable manner. If the distribution is non-degenerate, agents must take into account the whole distribution and its evolution when making consumption and saving decisions. My paper departs from these assumptions and tackles the consequences of accounting for the persistent heterogeneity observed in the data.

Second, my model is also related to other papers that study the role of heterogeneity in search-theoretic models of money. Molico (2006), Chiu and Molico (2010), Chiu and Molico (2011), Menzio, Shi and Sun (2013), Rocheteau, Weill and Wong (2021), and Chiu and Molico (2021) address this topic. Some of these papers explore the redistributive consequences of different money injection mechanisms. However, they do so in environments with only one asset and where the distribution of assets does not determine the incidence of monetary policy at the individual level. The numerical method I developed to solve the stationary equilibrium may be of independent interest. As opposed to Chiu and Molico (2011), the method does not require simulating a large number of agents to then fit a distribution using kernel density estimation methods. Likewise, in contrast to Rocheteau et al. (2021), my method is able to handle heterogeneity in both buyers and sellers so that we do not need to impose their types exogenously. My solution strategy relies on nonlinear methods and can accommodate more than one dimension in the agents' individual state space. I discuss the algorithm in detail in Appendix A.

Third, this paper contributes to the literature on OMOs and liquidity provision for the conduct of monetary policy.[3] Rocheteau, Wright and Xiao (2018) also study the effect

[2]Lagos, Rocheteau and Wright (2017) present a comprehensive review of the literature about New Monetarist models, where they summarize the existing work on liquidity, monetary policy, and monetary and credit arrangements.

[3]Some of the earlier work in this area includes papers such as Lucas (1972), Lucas (1984), Wallace (1981), Sargent and Smith (1987), Sargent and Wallace (1981), and Lucas (1990).

of OMOs and characterize the different types of equilibria that can emerge under several types of market structures and levels of assets' liquidity. They show that when assets have different levels of pledgeability and hence liquidity, OMOs can have real effects. However, to ensure their model's analytical tractability, Rocheteau et al. (2018) assume quasi-linear preferences, and therefore eliminate any eventual persistent heterogeneity among agents, rendering monetary policy absent of redistributional effects.

Kocherlakota (2003) examines the role of bonds and shows that having illiquid nominal risk-free bonds in monetary economies is essential, as they allow agents to engage in additional intertemporal exchanges of money. Similarly, Shi (2008) finds that there are efficiency gains of having this type of asset, as it provides an instrument for agents to (partially) self-insure against idiosyncratic liquidity shocks. Notably, Geromichalos, Licari and Suárez-Lledó (2007), Williamson (2012), Araujo and Ferraris (2020), Sterk and Tenreyro (2018), Rocheteau and Rodriguez-Lopez (2014), Carli and Gomis-Porqueras (2021) discuss the effect of OMOs, monetary policy and asset prices under different types of equilibria in economies without persistent heterogeneity.

Lastly, this paper is related to the literature that explores the role of household heterogeneity and portfolio choice in the transmission and implementation of monetary policy. This literature has used predominantly New Keynesian models where households that are subject to exogenous uninsurable idiosyncratic earning risk participate in centralized markets within cashless economies with nominal rigidities. This work focuses on studying the short-run stabilization properties of monetary policy, as opposed to the long-run effects in the presence of an explicit money demand I examine. Notably, besides reflecting expected return considerations, the portfolio choice in my model affects the surplus agents can extract in decentralized market meetings. Hence, households can, to an imperfect extent, influence their degree of risk exposition. As in my paper, in Kaplan, Moll and Violante (2018) and Bayer, Luetticke, Pham-Dao and Tjaden (2019), households save in two assets that differ in their degree of liquidity. However, in their papers, monetary policy is implemented via a Taylor rule without actively changing the public provision of liquidity.

Lee (2021) extends the framework of Kaplan et al. (2018) to allow for unconventional monetary policy in the form of quantitative easing. He finds that increasing the supply of liquid assets using quantitative easing has a non-monotonic effect on inequality as it operates by reducing unemployment while increasing the return on production capital. In my model, the effects of a permanent increase in the relative supply of illiquid assets reduce inequality and redistribute resources out of the wealthiest households in the economy because it attenuates the degree of market incompleteness. Gornemann, Kuester and Nakajima (2021) focus on stabilization policies and show that the design of a monetary regime can have sizable distributional implications as it directly affects the distribution of income risk. Similarly, Ravn and Sterk (2021) also propose a model where the distribution of income risk is endogenous and fluctuates along the business cycle, as precautionary savings put downward pressure on interest rates.

In contrast with these New Keynesian models with household heterogeneity, in my model, the distribution of assets matters in and of itself. This means that the importance of the distribution in determining equilibrium outcomes goes beyond pinning down aggregate quantities with its first moments. Agents in this economy actually care about the shape of the entire distribution as it determines decentralized market prices and shapes their liquidity risk. Importantly, this implies that the distribution of idiosyncratic risk is endogenous and that agents can influence their degree of risk exposition. My model also contributes to this literature in that it does not need to assume the presence of nominal rigidities for monetary policy to have real effects. Here, changes in monetary policy go beyond precautionary savings decisions and directly affect prices and the exposition to liquidity risk.

The remainder of the paper is organized as follows. Section 2 outlines the model economy. Section 3 describes the calibration strategy. Later, Section 4 presents the properties of the stationary equilibrium. Section 5 discusses the long-run properties of implementing a change in the composition of publicly provided liquidity using OMOs, and Section 6 contrasts the results in the baseline model with two alternative specifications. Later, Section 7 explores the mechanisms determining the welfare cost of inflation. Finally, Section 8 concludes.

2 Model

The model presented here is based in Lagos and Wright (2005) and Rocheteau et al. (2018), but with a few critical differences that will be discussed later. There is a continuum of agents of unitary mass. Time is discrete, agents live for infinite periods, and they discount time at the rate $\beta \in (0, 1)$. Each period is divided into two markets that operate sequentially: first, agents trade bilaterally in a decentralized market (DM), and later, they meet in a centralized market (CM). There is a unified fiscal and monetary authority that we will call government, which controls the supply of the two assets available to agents in this economy: fiat money and one-period nominal government bonds. Both of them are storable and perfectly divisible. Money has no intrinsic value, while government bonds represent claims of money in the next CM. Individual nominal money and bond holding will be normalized with respect to the beginning of the period money supply, M. Thus, if $\hat{m}$ and $\hat{a}$ are individual nominal money and bonds, respectively, then $m = \hat{m}/M$ and $a = \hat{a}/M$ denote individual relative money and bond holdings.

In the DM, agents may be randomly matched with others in bilateral meetings. Each agent specializes in producing a nonstorable differentiated good, c_d, that is "wanted" by other agents but not by that agent. This differentiated good can only be produced in the DM, and its technology of production is one-to-one on labor. The agents' period utility function is:

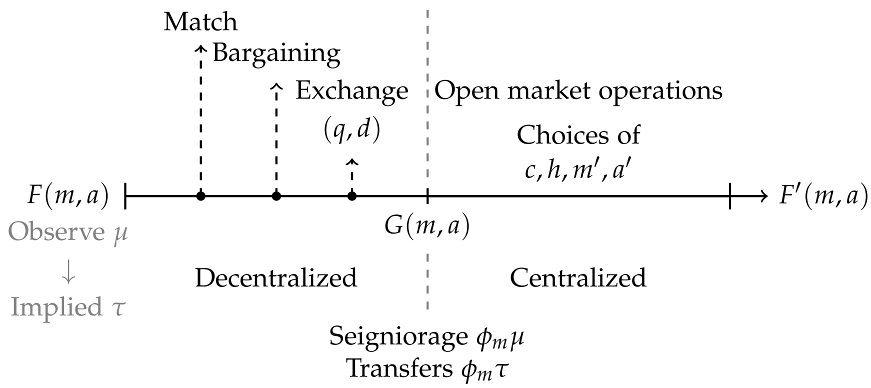
$$\mathcal{U}(c_d, h_d, c, h) = u(c_d) - v(h_d) + U(c, 1-h), \tag{1}$$

where u and v are, respectively, the utility function and cost function in the DM and U is the utility function in the CM. Here, c_d and h_d denote consumption and labor in the DM, while c and h are their counterparts in the CM (conversely, $1-h$ is leisure). The functions u and v satisfy: $u(0) = v(0) = 0$, $u' > 0$, $v' < 0$, $u'' < 0$, and $v'' \geq 0$; while U is twice continuously differentiable and strictly concave in both consumption and leisure. Unlike in Lagos and Wright (2005), here U is not restricted to be linear in h and, therefore, we will not necessarily have a degenerate distribution of assets.

When two agents are matched, we have two different types of meetings. If agent A wants what B produces but B does not want what A produces, we say we have a single-coincidence meeting where A is the buyer (consumer) and B is the seller (producer). We also have the possibility of not having any coincidence at all, where none of the matched agents like what the other produces.

Let α denote the probability of being matched, and let the probability of being matched as a buyer be $1/2$. When matched, the agents must determine the terms of trade between them: how much will be produced and at what price. Here, I assume that agents are anonymous, that their histories are private information, and that there is no record keeping. The only individual characteristics that are observable are asset holdings. However, bonds cannot be used in decentralized trading. These frictions prevent the existence of credit and make money essential as it is the only instrument they can use as a medium of exchange in meetings where barter is not an option (i.e., single-coincidence meetings). In this context, terms of trade are determined by a "take it or leave it" offer of the buyer to the seller.

Figure 1: Timing of Events



In the second subperiod, agents trade in a centralized (Walrasian) market. Agents have access to a production technology that transforms one unit of labor into one unit of a perishable homogeneous good. In this subperiod, agents can also adjust their asset holdings and receive lump-sum transfers from the government. The centralized market is also the time window used by the government to implement any change in its monetary policy involving OMOs. For example, if the government decides to increase its supply of money relative to government bonds, it will conduct a purchase of bonds that will, in turn, increase agents' money holdings. Of course, and this is a crucial point in this paper, agents entering the CM with different portfolios will be affected differently by this policy. Below, I present the model structure in more detail. See Figure 1 for an illustration of the exact timing of the model for a given period.

2.1 Government

The unified fiscal and monetary authority controls the supply of money, M, and nominal bonds, A. Let $K \equiv A/M$ denote the ratio of aggregate government bonds to aggregate money. Money grows at the constant rate μ^M, so $M' = (1 + \mu^M)M$, while bonds do so at the rate μ^A. I denote the state vector characterizing monetary policy as $\theta \equiv (K, \mu)$ where $\mu \equiv (\mu^M, \mu^A)$. Stationarity requires A to grow at the same rate of M, so that $\mu^M = \mu^A = \overline{\mu}$ and K is a constant. I also assume that the government maintains a balanced budget. Hence, it finances its debt service, lump-sum transfers to agents, T, and unproductive government expenditure, $\mathcal{G}$, with seigniorage and by issuing new one-period bonds. In this context, the government's balance is:

$$\phi_m K + T + \mathcal{G} = \mu^M \phi_m + \left(1 + \mu^A\right) \phi_m \phi_a K, \tag{2}$$

where ϕ_m and ϕ_a are the price of relative money and relative bonds, respectively. Note that the budget constraint in (2) is already expressed in terms of relative units.

Finally, government transfers (or taxes, if negative) and expenditure, are expressed in units of money. From (2), this implies that $T = \phi_m \tau$ and $\mathcal{G} = \phi_m g$, where

$$\tau + g = \mu^M - \left[1 - \left(1 + \mu^A\right) \phi_a\right] K. \tag{3}$$

There are multiple ways for the government to balance its budget using transfers and expenditures; thus, I write the model in the most general form. In the quantitative analysis, I solve two ways of closing the model: (i) a case with positive government expenditure, $g > 0$ and $\tau = 0$, and (ii) a case with positive lump-sum transfers, $g = 0$ and $\tau > 0$. Nevertheless, the first case with zero lump-sum transfers offers a better approximation to study the effects of monetary policy changes via household levels of wealth and portfolio composition rather than via the traditional lump-sum transfers.

Note that the real interest rate on public debt is given by:

$$1 + r = \frac{1}{\phi_a}. \tag{4}$$

Also, by the Fisher equation, the nominal counterpart of r is $(1 + i) = (1 + r)(1 + \pi)$, where π denotes the inflation rate, which in the stationary equilibrium satisfies $\pi = \mu^M$. These definitions are helpful in calibrating the model and interpreting its results. Importantly, when discussing the effects of OMOs, I consider the case of the fiscal-monetary authority using these operations to target a particular level of this nominal interest rate on bonds, similar to Rocheteau et al. (2018).

2.2 Decentralized Market

The probability measures $F(m, a)$ and $G(m, a)$ summarize the distribution of agents over m and a for the DM and CM, respectively. These distributions are defined on the Borel algebra, $\mathcal{Z}$, generated by the open subsets of the space, $Z = \mathcal{M} \times \mathcal{A}$. Since the probability of being matched with someone with a particular level of money and bonds depends on how many of those agents are in the entire population, the probability distribution is the aggregate state variable. These distributions evolve according to $G = \Gamma_G(F, \theta)$ and $F' = \Gamma_F(G, \theta)$. Given this, and the policy implemented by the monetary authority, in equilibrium we have:

$$\int_{\mathcal{M}\times\mathcal{A}} mdF\left([dm \times da]\right) = 1 \tag{5}$$

$$\int_{\mathcal{M}\times\mathcal{A}} adF\left([dm \times da]\right) = K \tag{6}$$

Let $\tilde{m}$ represent money holdings at the end of the DM and define $x \equiv \tilde{m} + a + \tau$ as wealth. Given that bonds are nominal, and that government transfers are also expressed in terms of money, x is the only relevant individual state variable for agents. Now, let $V\left(m, a; F, \theta\right)$ and $W\left(x; G, \theta\right)$ be the value functions at the beginning of the DM and CM, respectively. Since we are only considering single-coincidence meetings, we have no situations where barter is an option to conduct the exchange process. I also assume that, since bonds are illiquid, money is the only acceptable asset when exchanging in any of the goods markets. When two agents are randomly matched, one as a buyer and the other as a seller, they decide the terms of trade. I assume that the buyer makes a "take it or leave it" offer to the seller: the buyer offers to buy q of the differentiated good at the price d.

In a meeting where the buyer's portfolio is $z_b = (m_b, a_b)$ and the seller's is $z_s = (m_s, a_s)$, terms of trade are determined according to the following problem:

$$\max_{q,d} \; u\left(q\right) + W\left(m_b + a_b + \tau - d; G, \theta\right), \tag{7}$$

subject to the seller's participation constraint

$$-v\left(q\right) + W\left(m_s + a_s + \tau + d; G, \theta\right) \geq W\left(m_s + a_s + \tau; G, \theta\right), \tag{8}$$

to the law of motion of the distribution $G\left(m\right) = \Gamma_G\left(F\left(m\right), \theta\right)$, and to $0 \leq d \leq m_b$, $q \geq 0$. Note that the continuation values of buyers and sellers take into account their money and bond holdings when exiting the current DM and the transfer they will receive at the beginning of the next CM. For each meeting of type (m_b, a_b, m_s, a_s), and given an aggregate state $\{F\left(m, a\right), \theta\}$, we have that the terms of trade $q\left(z_b, z_s\right) \equiv q\left(m_b, a_b, m_s, a_s; F, \theta\right)$ and $d\left(z_b, z_s\right) \equiv d\left(m_b, a_b, m_s, a_s; F, \theta\right)$ solve the problem stated above.

In this context, the expected lifetime utility at the beginning of the DM of an agent with portfolio (m, a)—i.e., before knowing if they are matched or not, and before knowing what their role in an eventual match would be—is given by the following functional equation:

$$
\begin{aligned}
V(m,a;F,\theta) = &\frac{\alpha}{2}\int_{\mathcal{M}\times\mathcal{A}}\left\{u\left(q\left(z,z_s\right)\right)+W\left(m-d\left(z,z_s\right)+a+\tau;G,\theta\right)\right\}F\left(d\left[m_s\times a_s\right]\right)\\
&+\frac{\alpha}{2}\int_{\mathcal{M}\times\mathcal{A}}\left\{-v\left(q\left(z_b,z\right)\right)+W\left(m+d\left(z_b,z\right)+a+\tau;G,\theta\right)\right\}F\left(d\left[m_b\times a_b\right]\right)\\
&+(1-\alpha)\,W\left(m+a+\tau;G,\theta\right),
\end{aligned}
\tag{9}
$$

where, as noted before, $G(m,a) = \Gamma_G\left(F(m,a),\theta\right)$. Here, the first term is the expected value of being matched as a buyer (taking into account that there are several "types" of sellers they can meet with), the second term is the expected value of being matched as a seller, and the last term is the value of not being matched or being matched in a no-coincidence meeting.

2.3 Centralized Market

Recall that agents enter the CM with a certain level of wealth, a variable that includes nominal assets at the end of the previous DM (i.e., net of decentralized trading payments) and government transfers. Thus, an agent with wealth x at the beginning of the CM has lifetime utility given by:

$$
W(x;G,\theta) = \max_{c,h,m',a'}\left\{U(c,h)+\beta V\left(m',a';F',\theta'\right)\right\} \tag{10}
$$

subject to

$$
c = h+\phi_m(G,\theta)\,x-\phi_m(G,\theta)\left[m'+\phi_a(G,\theta)\,a'\right]\left(1+\mu^M\right) \tag{11}
$$

$$
F'\left(m',a'\right) = \Gamma_F\left(G(m,a),\theta\right). \tag{12}
$$

2.4 Equilibrium

A monetary equilibrium is a set of functions for value $\{V(m,a;F,\theta), W(x;G,\theta)\}$, allocations $\left\{c(x;G,\theta), h(x;G,\theta), m'(x;G,\theta), a'(x;G,\theta), q(z_b,z_s)\right\}$, prices $\left\{d(z_b,z_s), \phi_m(G), \phi_a(G)\right\}$, and distributions $\left\{F(m,a),\ G(m,a)\right\}$ such that, given the policy θ:

1. Values $V(m,a;F,\theta)$ and $W(x;G,\theta)$ and decision rules $c(x;G,\theta)$, $h(x;G,\theta)$, $m'(x;G,\theta)$, $a'(x;G,\theta)$ satisfy the definitions above, for any given $\{q,d,\phi_m,\phi_a\}$ and $\{F(m,a), G(m,a)\}$.

2. Terms of trade $\{q(z_b,z_s), d(z_b,z_s)\}$ in the DM solve the problem in (7) given $V(m,a;F,\theta)$ and $W(x;G,\theta)$.

3. There is a monetary equilibrium: $\phi_m > 0$.
4. The government has a balanced budget. This is, Equation (2) holds.
5. The money and bond markets clear, i.e., Equations (5) and (6) are satisfied.[4]
6. The law of motions for $F(m,a)$ and $G(m,a)$ are given by $\Gamma_F(\cdot)$ and $\Gamma_G(\cdot)$, respectively. These maps are consistent with the initial conditions and the evolution of money holdings implied by DM and CM trade. This is,

$$\begin{aligned} G(m,a) = \Gamma_G(F,\theta) = &\frac{\alpha}{2}\int_{\{z_s \in Z\}}\int_{\{(m-d(z,z_s)+\tau,a)\in Z\}} F([dm \times da])\, F([dm_s \times da_s]) \\ &+ \frac{\alpha}{2}\int_{\{z_b \in Z\}}\int_{\{(m+d(z_b,z)+\tau,a)\in Z\}} F([dm \times da])\, F([dm_b \times da_b]) \\ &+ (1-\alpha)\int_{\{(m+\tau,a)\in Z\}} F([dm \times da]) \end{aligned} \tag{13}$$

and

$$F'(m,a) = \Gamma_F(G,\theta) = \int_{\{(m'(x),a'(x))\in Z\}} G([dm \times da]), \tag{14}$$

where, as before, $z_i = (m_i, a_i)$ for $i = b, s$ and $Z = \mathcal{M} \times \mathcal{A}$.

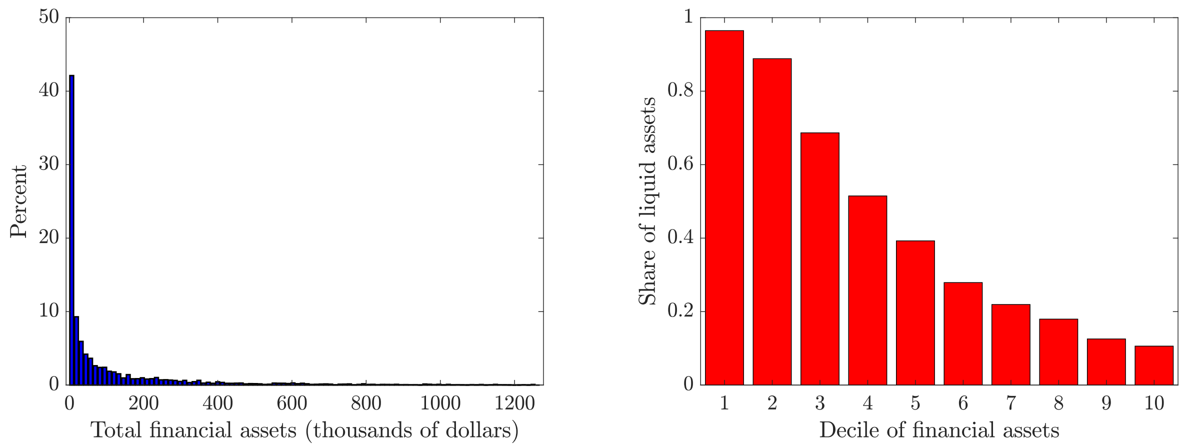
3 Data and Calibration

The quantitative results of the model regarding the effectiveness of changes in monetary policy in the long run depend on how closely the model resembles the distribution of financial assets in the data. In the same spirit, it is necessary to match agents' portfolio composition along the distribution of assets. Figure 2(a) presents the distribution of financial assets computed using data from the SCF for 2007. This distribution exhibits a long right tail, and there is a large concentration of households at lower levels of financial assets. Consequently, the Gini coefficient for this distribution is 0.86. Figure 2(b) shows the share of financial assets held in liquid assets by decile. The most asset-poor households (decile 1) tend to hold close to 95 percent of their financial assets in liquid instruments. In comparison, the most asset-rich households only hold slightly more than 10 percent in liquid assets on average.

[4] Eq. (4) shows that there is a one-to-one mapping between the price of nominal bonds, ϕ_a, and the nominal interest rate, i. This, together with the market clearing condition in the bonds market, implies that there is a direct relation between i and the relative supply of bonds, K. Thus, in the stationary equilibrium, for any policy $\theta = (K, \mu)$, there is an equivalent policy (i, μ). This result implies that the government can choose both a nominal interest rate and an inflation rate, while the price and the supply of bonds adjust to satisfy market clearing in a way that preserves consistency with the Fisher equation.

Figure 2: Distribution and Composition of Financial Assets in the SCF 2007

(a) Distribution of Financial Assets **(b)** Share Held in Liquid Assets



Notes: Data from the Survey of Consumer Finances for 2007. Liquid assets computed as the sum of money market, checking, savings and call accounts, and prepaid cards. Financial assets include all liquid assets plus certificates of deposit, directly held pooled investment funds, saving bonds, directly held stocks and bonds, cash value of whole life insurance, other managed assets, quasi-liquid retirement accounts, and other financial assets. The share of liquid assets is computed as the ratio of liquid assets to total financial assets.

The model is parameterized such that one model period is equivalent to one quarter. Hence, the discount factor is set to $\beta = 0.99$. Aggregate equilibrium objects simultaneously target averages for the period 1984Q1–2007Q4. The money growth rate is consistent with an annual inflation rate, in the stationary equilibrium, equal to the average of 3.1 percent in the data. To measure money in the data, I follow Alvarez, Atkeson and Edmond (2009), where money is computed as the sum of currency, demand deposits, saving deposits, and time deposits. For the period of study, the ratio of the total federal debt to money is 1.28. In the model, this is equivalent to K. Regarding interest rates for government debt, I use the secondary market rate of the 3-month Treasury bills. The average of this annual nominal rate for the same period is 4.9 percent.

I follow Lagos and Wright (2005) for the utility and cost functions in the DM:

$$u\left(c_d\right) = \frac{1}{1-\eta}\left[\left(c_d + b\right)^{1-\eta} - b^{1-\eta}\right] \tag{15}$$

$$v\left(h_d\right) = B h_d^{\nu} \tag{16}$$

with $\eta > 0$, $B > 0$, $\nu > 0$, and $b \approx 0$. However, for the CM, I drop the quasi-linearity assumption. In particular, the utility function in the CM is concave in consumption, has a constant Frisch elasticity of labor supply, and is given by:

$$U(c, h) = \log c - \kappa \frac{h^{1+\chi}}{1+\chi}, \tag{17}$$

where χ is the inverse of the Frisch elasticity of labor supply and $\kappa > 0$ a scale parameter. The parameter that governs the curvature of the utility function in the DM, η, is set close to 1, so the utility function is close to being logarithmic.

The parameters determining the scale and curvature of the cost of working in the DM, B and ν, the scale of the disutility of labor in the CM, κ, the Frisch elasticity of labor supply, χ, and the probability of being matched in a DM meeting, α, are jointly calibrated to match (i) the ratio of the total federal debt to money, (ii) the average markup, (iii) the velocity of money, (iv) labor in the CM being one-third, and (v) the semi-elasticity of money demand with respect to the nominal interest rate. In what follows, I discuss the calibration of the model with no transfers, $\tau = 0$, and positive government expenditure, $g > 0$, but the moments generated by the same parameter values in the model with $\tau > 0$ and $g = 0$ are almost identical. The Frisch elasticity of labor supply is set equal to 2.22, which means that $\chi = 0.45$. This elasticity is in the range of what Chetty, Guren, Manoli and Weber (2011) document. Finally, the probability of having a DM meeting is calibrated to $\alpha = 0.625$, which means that agents can participate in decentralized market meetings every 1.6 quarters or 4.8 months, on average.

The model is able to replicate a velocity of money close to the one observed in the data for the sample period: 2.18 in the model versus 2.25 in the data. Moreover, the average markup[5] generated in DM meetings (32 percent), is consistent with the 30 percent average markup reported in Faig and Jerez (2005), while $K = 1.32$ and the semi-elasticity of money demand with respect to the nominal interest rate is -0.058, close in range to the one documented in Lucas (2000), Aruoba, Waller and Wright (2011), and Berentsen, Menzio and Wright (2011). Table C1 in Appendix C presents the local elasticities of some of the model's moments with respect to the parameters of this baseline calibration. Table 1 summarizes the remaining parameters, while Table 2 reports the targeted moments in the data and their model counterparts.

[5]The markup is computed as the ratio of the price and the marginal cost of production of the quantities agreed in a given meeting. The average markup weighs the set of possible individual markups for the occurrence probabilities of those meetings.

Table 1: Parameter Values

Parameter		Value
Internally calibrated parameters		
α	Probability of meeting	0.625
χ	Inverse of Frisch elasticity	0.45
κ	Scale of disutility of labor	5.15
B	Scale of cost of working	0.23
ν	Curvature of cost of working	1.47
Externally calibrated parameters		
β	Discount factor	0.99
$\overline{\mu}$	Growth rate of money and bonds	0.0077
η	Curvature of utility of consumption	0.99
b	Scale parameter in $u(c_d)$	0.0001

Notes: Internally calibrated parameters are simultaneously calibrated to match the moments reported in Table 2. The growth rate of money and bonds is consistent with an average annual inflation rate of 3.1 percent for the period 1984Q1–2007Q4.

Table 2: Calibrated Moments

Simultaneous calibration targets	Data	Model
Velocity of money	2.25	2.18
Hours worked in CM	0.33	0.33
Average markup in DM	0.30	0.32
Semi-elasticity of money demand	-0.064	-0.058
Ratio of total federal debt to money	1.28	1.32

Notes: Velocity of money and the ratio of total federal debt to money in the data are averages for the period 1984Q1–2007Q4. Money is computed as the sum of currency, demand deposits, saving deposits, and time deposits. The average markup in DM meetings is from Faig and Jerez (2005) and the semi-elasticity of money demand is taken from Aruoba et al. (2011).

Numerical Method

The numerical method employed here to solve for the stationary equilibrium has not, to my knowledge, been used in the literature on search-theoretic models of money. Although most papers in this area use assumptions that guarantee analytical tractability, the few that implement numerical methods to account for the intertemporal heterogeneity in money holdings employ solution strategies that are costly in terms of efficiency, that may sacrifice accuracy, and that are not easily extended to higher dimensions. For example, the numerical method in Molico (2006) and Chiu and Molico (2010, 2011) require simulating matches between agents and performing a kernel density estimation of the distribution of money with these observations. Given that we must consider every possible meeting, the size of the simulation has to be large enough to have several observations for each match. This also implies that the required number of simulated matches increases with the dimension of the portfolio, as the number of possible matches may increase. In addition, having to use kernel estimation may imply losing accuracy in the estimated distribution. Chiu and Molico (2021) parameterize the distribution of money, while Jin and Zhu (2019) impose imperfectly divisible money and lotteries for computational tractability.

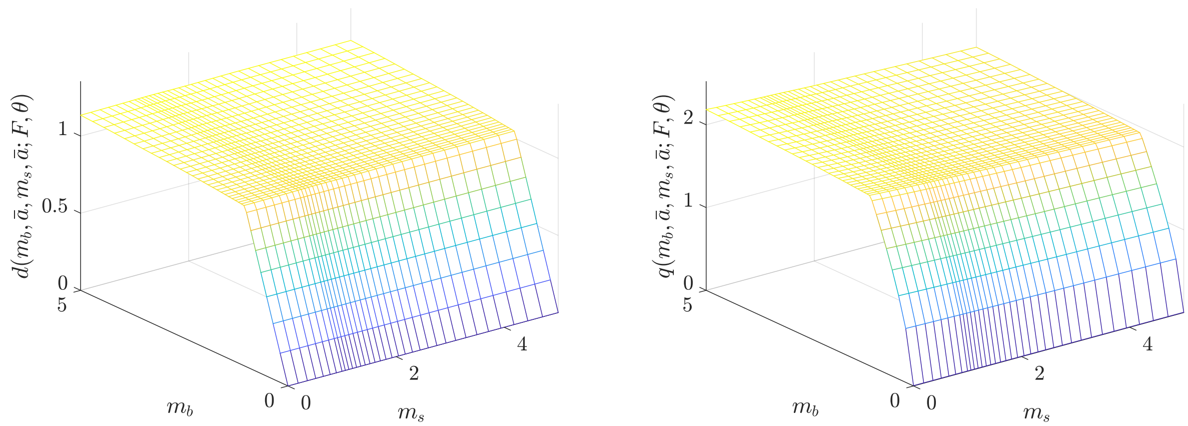
My solution method employs a nested fixed-point algorithm that solves iteratively for the agents' decision rules, the terms of trade in decentralized trading, the level of lump-sum transfers or government expenditure consistent with the government balance, and the prices that clear the markets of bonds and money. The solution method requires repeated applications of the contraction mapping implied by (9)-(12), for some given terms of trade. With the household decision rules in hand, we can then iterate over the distributions of assets in the DM and CM until convergence. For this step, it is necessary to store the distribution of assets using fine grids defined over bonds and money. After guaranteeing that both asset markets clear, an outer loop solves the terms of trade of DM meetings that are consistent with households' value function (10). Importantly, these terms of trade solve a two-sided bargaining problem on the money and bond grids for buyers and sellers, i.e,. on the discrete space $\mathcal{Z} \times \mathcal{Z}$, where $\mathcal{Z} = \mathcal{M} \times \mathcal{A}$. A detailed description of the numerical method can be found in Appendix A.

4 Properties of Stationary Equilibrium

For this section, I consider the case of no transfers, $\tau = 0$, and positive government expenditure, $g > 0$. The converse case with $\tau > 0$ and $g = 0$ yields very similar results

Figure 3: Terms of Trade: Monetary Payments and Quantities

(a) Payments **(b)** Quantities



Notes: For expositional purposes, the figures only present terms of trade as a function of (m_b, m_s) for meetings between both buyers and sellers with average bond holdings, $\bar{a}$. See Figure C1 in Appendix C for more figures emphasizing the role of bond holdings in determining terms of trade in the decentralized market.

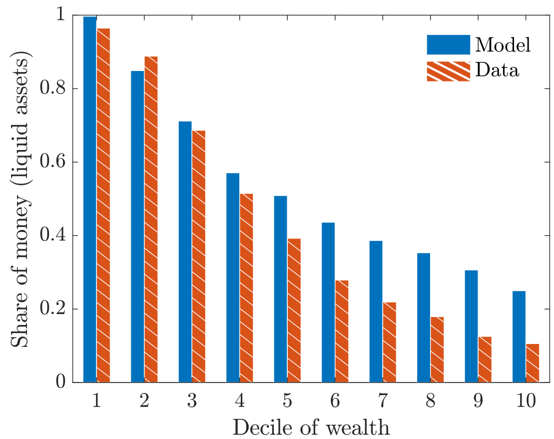
with respect to terms of trade, distribution of assets, and decision rules. Before examining the role of monetary policy changes, I first characterize the stationary equilibrium by showing how the idiosyncratic liquidity risk that arises from decentralized trading shapes terms of trade and the distribution of assets.[6]

Figure 3 shows the terms of trade for quantities and monetary payments exchanged in meetings between buyers and sellers with average bond holdings. The figure depicts terms of trade as a function of the buyer's money holdings, m_b, and the seller's, m_s. In any possible meeting, the quantities exchanged in the DM are an increasing function of m_b and depend negatively on m_s. In addition, monetary payments increase in both m_b and m_s. Intuitively, the wealthier the seller is, the higher is his opportunity cost of working. Hence, the seller requires more money in exchange for a lower level of output in order to remain indifferent about trading. On the other hand, the buyer's willingness to pay increases with her money holdings as long as it guarantees her higher consumption, i.e., the opportunity cost of spending an extra dollar in DM trading decreases with m_s. There is a similar outcome in terms of bond holdings. Wealthier buyers are willing to pay more, and sellers with higher bond holdings have a higher opportunity cost.[7]

[6] For all these numerical experiments, I use the solution method outlined in Appendix A.

[7] See Figure C1 in Appendix C for a summary of the quantities traded, monetary payments, and prices per unit for more types of meetings in which bond-rich and bond-poor agents meet.

Figure 4: Share of Liquid Assets in Portfolio



Notes: Shares of liquid assets in agents' portfolios by decile in the model's stationary equilibrium and in the SCF 2007 data. In the model, the share of liquid assets is computed as $m'/(m'+a')$.

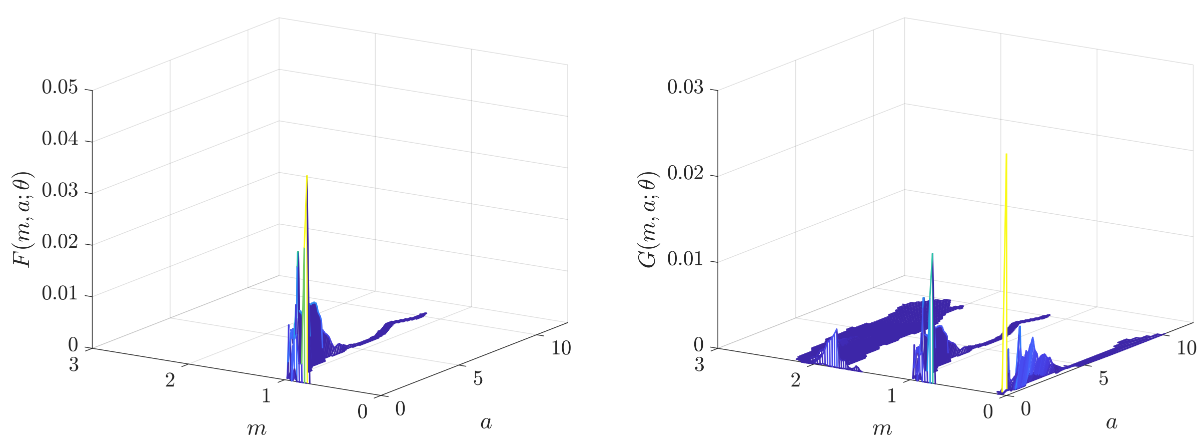
Note that, as is standard in search-theoretic models of money, trading frictions in decentralized trading generate inefficient allocations. The efficient level for quantities exchanged, q^*, solves $u'(q^*) = v'(q^*)$. However, differences in total wealth and not only differences in money holdings are the ones giving rise to inefficient outcomes.[8] For the benchmark calibration of the model, $q^* = 2.1$. Thus, as shown in Figure 3(b) for the case of agents with average bond holdings, meetings in which a wealthy buyer trades with a poor seller result in an inefficiently high level of production. Conversely, meetings with poor buyers tend to generate an inefficiently low level of trade. This means that a higher dispersion in money and bond holdings generates a more inefficient equilibrium.

The various possible outcomes in decentralized trading lead to buyers and sellers entering the next CM with different asset holdings and portfolio compositions. As opposed to models where preferences are quasi-linear, wealth effects play a significant role in determining agents' decisions in the CM, and not all agents choose to accumulate the same amount of money or bonds (see Figures C2 and C3 in Appendix C). As a consequence, the heterogeneity intrinsically generated by decentralized trading is persistent across periods.

[8] Let γ be the multiplier on the buyer's liquidity constraint and define wealth as $x_i = m_i + a_i + \tau$ for $i = b, s$. The optimality conditions for the bargaining problem imply $u'(q)/v'(q) = [W'(x_b - d) + \gamma]/W'(x_s + d)$. Given that W is monotonically increasing and concave, if $x_b < x_s$, then $W'(x_b - d) > W'(x_s + d)$, which implies that $u'(q) > v'(q)$ and $q < q^*$.

Figure 5: Distribution of Assets

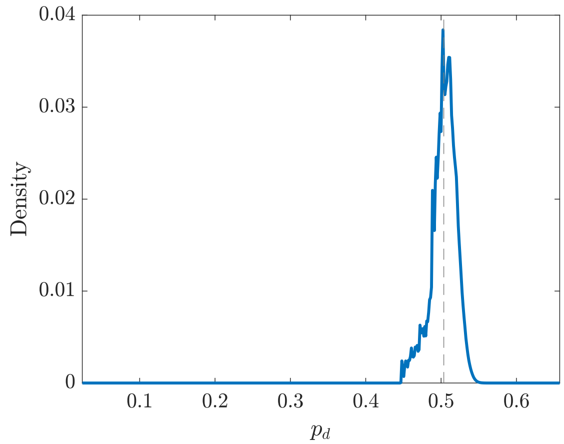
(a) At the Beginning of DM **(b)** At the Beginning of CM



Interestingly, this mechanism generates a composition of portfolios consistent with the data presented before in Figure 2(b). Figure 4 presents the share of total assets held as money, along the different levels of wealth (total assets or cash). This figure shows how asset-poor agents prioritize the accumulation of money over bonds as they prepare to enter a new round of decentralized trading (see Figure C2). On the other hand, asset-rich agents can self-insure more easily by purchasing a higher share of bonds.

Figures 5(a) and 5(b) depict the distribution of agents over money and government bonds at the beginning of the DM and the CM. The distribution at the beginning of the DM is relatively concentrated in terms of money holdings but disperse in bonds. In contrast, the distribution entering the CM spreads agents out with respect to their money holdings depending on if they are buyers, sellers, or unmatched in the previous DM. Why is there such a drastic change in the dispersion of money but not in the dispersion of bonds? We are observing risk-averse agents using bonds to partially self-insure against the idiosyncratic liquidity shocks they may experience in the DM. However, the reshuffling of a poor agent's portfolio is not immediate because the marginal cost of working is increasing in the hours worked. As a result, agents adjust their money balances very promptly because of the risk of being matched as buyers in the next period. Moreover, agents accumulate bonds at a slower pace after having reached some level of money that is enough to face a liquidity shock in the next DM. This pattern is clear when we observe the decision rules for $m' = g_m(x; G, \theta)$ and $a' = g_a(x; G, \theta)$ in Figure C2 in Appendix C.

Figure 6: Stationary Distribution of Prices

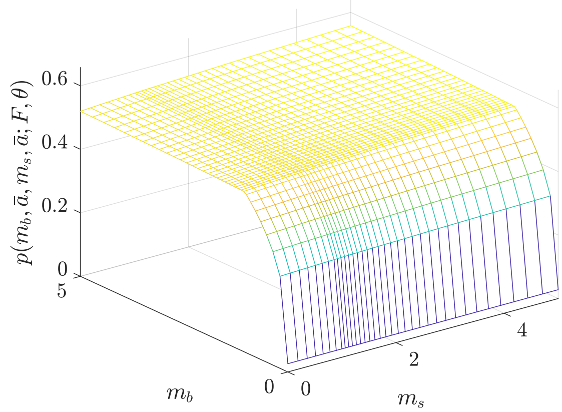


Notes: For a particular meeting with buyer's and seller's portfolios $z_b = (m_b, a_b)$ and $z_s = (m_s, a_s)$, respectively, prices are defined as $p(z_b, z_s) = d(z_b, z_s) / q(z_b, z_s)$. The distribution of prices accounts for all possible meetings between buyers and sellers given the stationary distribution of agents over money and bond holdings.

This heterogeneity across agents' portfolios when entering the DM generates a fully-fledged distribution of prices, where the price per unit is defined as $p_d = d/q$. Figure 6 shows such distribution of prices. Given the relatively high concentration of agents around a particular value of m, there is a large fraction of meetings with a price around the mean price (0.5). However, the variance in bond holdings is sufficient to generate a standard deviation of 0.017 in the observed prices. As with the terms of trade for quantities and payments, in Figure 7, I compute prices per unit as a function of both agents' money holdings while fixing their stock of bonds at the average level. One of the most relevant variables when determining prices is how costly it is for the seller to produce. A seller with high levels of money and bond holdings requires exchanging at higher prices in decentralized trading because of his participation constraint. The fact that, in equilibrium, the model exhibits a large dispersion in asset holdings explains the variance in the distribution of prices. Hence, as long as the distribution of money and bonds has a sluggish response to economy-wide shocks (e.g., OMOs), we can expect that the distribution of prices will react accordingly.[9]

[9] If agents do not take into account the whole distribution when computing Eq. (9), they would be miscalculating the size of the DM liquidity shocks. In particular, the maximum errors in payments range from 0.36 for the more asset-poor households to 0.30 for the wealthiest households. The portfolio of the average household is $\overline{m} + \overline{a} = 2.32$, so these errors represent roughly 14 percent of their total wealth. As discussed before, given that terms of trade improve with wealth, asset-rich households have a lower cost of miss-forecasting future terms of trade.

Figure 7: Terms of Trade: Prices



Notes: Possible outcomes for meetings between buyers and sellers with average bond holdings, $\overline{a}$. Prices are defined as $p\left(z_b, z_s\right) = d\left(z_b, z_s\right)/q\left(z_b, z_s\right)$, where buyers' and sellers' portfolios are given by $z_b = (m_b, \overline{a})$ and $z_s = (m_s, \overline{a})$, respectively.

5 The Effects of Monetary Policy Changes in the Long Run

Now I turn to investigate the consequences of changes in monetary policy in the long run. First, I examine the stationary equilibrium of the model under different targeted nominal interest rates. As discussed before, each level of the targeted rate requires the consolidated fiscal and monetary authority to adjust the public provision of assets. I assume that the government implements a particular level of $K = A/M$ by conducting a one-time permanent OMO and then study the long-run properties of such a policy. Second, I assess how these stationary equilibria change for different trend inflation values, $\overline{\mu}$.

5.1 Open Market Operations in the Long Run

Table 3 presents the percent changes in some macroeconomic aggregates for two different stationary equilibria after changes in the nominal interest rate with respect to the one in the baseline economy. The results consider two alternative ways of closing the model. For both of these cases, the baseline calibration in Section 4 is where the targeted annual rate is 4.9 percent; while the two additional stationary equilibria consider targeted rates 1 percentage point (p.p.) annual above and below the baseline level. As suggested by the theoretical models of Kocherlakota (2003) and Shi (2008), for both cases, implementing higher interest rates on public debt requires an increase in the relative supply of government bonds. This is simply a natural response from the demand for bonds: for a given level of the trend inflation, an increased supply of bonds pushes their price down, which in turn increases the nominal interest rate. As a consequence, higher nominal rates are associated with increased overall liquidity in the economy.[10] Meanwhile, when interest rates are high, the relative scarcity of money translates into an increased price of money. Notably, the differences in liquidity needed to target a 1 p.p. change in the nominal interest rate are larger in the zero lump-sum transfer case, as apposed to the $\tau > 0$ case given that the government is injecting additional liquidity evenly across all households in the latter.

The increased overall liquidity is associated with more economic activity. In particular, and focusing on the $g > 0$ case, while there is a slight change in the CM's level of activity, output in the DM falls -1.1 percent when the annual interest rate goes, permanently, from 4.9 to 3.9 percent and increases 0.5 percent when we compare the benchmark scenario with the case where the interest rate is 5.9 percent. The relatively small increase in the CM's production for higher levels of aggregate liquidity comes from the fact that the wealth effect associated with a higher disposable income is partially canceled out—in the aggregate level—by the substitution effect originating from the increased prices of money and nominal bonds.

In contrast, aggregate output from DM meetings increases with increased total liquidity levels because wealth becomes more evenly distributed. While the dispersion in bond holdings increases, this occurs in a smaller proportion than the rise in the relative supply of bonds. Likewise, given the increased supply of assets that let agents hedge their idiosyncratic risk, and the subsequent drop in the price of public debt, it becomes more attractive for agents to have a portfolio with lower participation of money. This explains the reduced dispersion in money holdings in the DM, as well as in the CM. This contraction in both dimensions of wealth reduces the inefficiencies in DM trading discussed earlier so that any pair of agents meeting in the DM look more alike and exchange more DM goods at lower and less disperse prices.

These results also highlight the presence of nonlinearities. Compared with the baseline scenario, in the case with $g > 0$, targeting an interest rate of 1 p.p. higher or lower requires a change in the relative supply of bonds of 23.6 and -15.2 percent, respectively. Total output contracts -0.31 percent in the economy with reduced liquidity, significantly

[10] Total liquidity is defined as the sum of the nominal value of the aggregate supply of money and government bonds. This is $1 + \phi_a K$.

Table 3: Percent Changes in Stationary Equilibrium for Given Changes in the Nominal Interest Rate

	Positive gov. expenditure		Positive transfers	
Nominal interest rate	−1p.p.	+1p.p.	−1p.p.	+1p.p.
Ratio bonds to money	-15.19	23.59	-16.11	21.74
Total output	-0.31	0.16	0.04	0.00
Output in DM	-1.09	0.47	-0.08	0.08
Output in CM	0.00	0.03	-0.03	0.03
Std. dev. money DM	7.55	-15.84	16.25	-14.99
Std. dev. money CM	-1.32	0.42	-1.28	0.52
Std. dev. bonds	-9.76	16.64	-12.46	16.05
Average DM price	-1.19	0.12	-1.23	0.36
Std. dev. DM price	0.60	-8.98	2.41	-8.43
Transfers	–	–	20.0	-24.0

Notes: Percentage change in stationary equilibria moments for two different targeted nominal interest rates with respect to the baseline calibration and for both ways of closing the model. In each case, the baseline scenario targets an annual 4.9% rate, while the two additional scenarios target a 1 p.p. change relative to the baseline rate (3.9% and 5.9%). In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$.

more (in absolute value) than the increase of 0.16 percent for the case with a higher supply of bonds. Most of this difference is explained by the changes in DM activity. As agents have fewer chances of self-insuring, there is more risk associated with bilateral meetings and an increased incidence of inefficiencies in DM trading. A similar mechanism also operates in the case with positive transfers.

Despite the relatively small changes in the CM's activity at the aggregate level, there are significant differences between these stationary equilibria along the distribution of agents. Table 4 shows the long-run changes in CM consumption, labor, and asset holdings, as well as DM consumption, for different groups of agents divided by wealth, after a one-time permanent OMO that targets a higher interest rate, for both ways of closing the model.[11]

The poorest agents gain the most, in the long run, when the economy moves to a scenario of increased liquidity with higher interest rates. Agents below the 50th percentile consume more and work less in the CM. Similarly, agents in the two lowest quartiles increase their money holdings, while agents between the 10th and 50th percentiles also

[11]See Table C2 in Appendix C for the long-run results of an OMO that lowers the annual interest rate in 1 p.p, for both cases of the model.

experience higher growth in their bond holdings than those at the top half of the distribution.[12] Note that it is still optimal for agents in the lowest decile to have a portfolio with no bonds in it.

The reduced concentration of assets also benefits the agents at the lowest end of the wealth distribution in their DM meetings. As shown in Table 3, agents trade more at lower and less dispersed prices. This implies that, primarily, agents in the lowest half of the distribution obtain better terms of trade and are able to consume more when they are buyers in bilateral meetings.[13]

Importantly, when the government injects money via lump-sum transfers, there is less redistribution towards the lower end of the distribution of wealth. Consumption on both markets, as well as money and bond holdings increase to a lesser extent for asset-poor households with $\tau > 0$. This effect is explained by the government partially offsetting some of this redistribution when giving all households the same lump-sum transfer. In contrast, when $\tau = 0$ and $g > 0$, the endogenous effects that follow the increased liquidity have a stronger impact on asset-poor households who are characterized by having higher marginal utilities, and who would benefit more from having a stronger bargaining position in DM meetings. For this reason, in what follows, I focus on the zero lump-sum transfer case.

5.2 The Role of Trend Inflation

The level of trend inflation also matters when determining the long-run effects of changes in monetary policy at the aggregate level, as well as at the distributional one. Table 5 compares key macroeconomic variables for different stationary equilibria with annual interest rates of 4.9 and 5.9 percent and annual trend inflation levels of 2.1 and 3.1 percent, for the case with $\tau = 0$ and $g > 0$.[14] When money and bonds grow at a faster pace,

[12]The higher supply of government bonds is distributed across almost the entire population. All agents, except the ones in the first decile, increase their bond holdings.

[13]While the prevalence of trading frictions, captured by the probability of having a DM match, α, affects the equilibrium supply of bonds needed to target a given interest rate, it does not significantly impact the redistributional effects of an OMO. Under an alternative version of the model solved with $\alpha = 1$, the redistribution following an OMO that increases rates is very similar to the baseline case in Table 4. The average difference in absolute value between the two cases is only 0.13 percentage points. With a higher chance of having a DM meeting, households demand more bonds for self-insurance purposes. This raises the dispersion of bond holdings, making households more dissimilar and raising markups. Hence, having an increased supply of assets for insurance is offset by the more unequal distribution of those assets, resulting in similar redistributional effects.

[14]Recall that the baseline economy is calibrated with a level of annual trend inflation of 3.1 percent and a annual nominal interest rate of 4.9 percent.

Table 4: Long-Run Distributional Effects of an Open Market Operation from $i = 4.9\%$ to $i = 5.9\%$

Percentile	≤ 10	**10-25**	**25-50**	**50-75**	**75-90**	≥ 90
Positive gov. expenditure						
Consumption (CM)	1.11	0.32	-0.22	0.27	-0.53	-0.71
Labor	-2.36	-0.70	0.49	-0.62	1.19	1.63
Money holdings	3.05	1.87	0.12	0.21	-0.28	-3.77
Bond holdings	–	63.12	32.14	20.86	19.78	19.53
Consumption (DM)	2.56	2.29	0.38	0.27	0.00	-1.94
Positive transfers						
Consumption (CM)	0.93	0.39	-0.33	0.16	-0.35	-0.88
Labor	-1.97	-0.84	0.72	-0.37	0.78	2.01
Money holdings	2.66	1.38	0.00	1.12	-1.39	-3.49
Bond holdings	–	59.50	30.47	18.91	18.82	19.00
Consumption (DM)	2.50	1.76	-0.01	0.31	-1.03	-1.98

Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates for both ways of closing the model. Changes are with respect to the baseline economy with an annual 4.9% nominal interest rate. In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$. Agents in the lowest decile have zero bond holdings in both stationary equilibria.

assets lose value more rapidly, and agents trade at higher prices in their DM meetings. This also implies that agents are in higher need of self-insuring against the idiosyncratic liquidity shocks they may experience in DM trading. Thus, the increased demand for bonds lowers the supply needed to implement a given nominal interest rate. Therefore, an economy with higher trend inflation has less liquidity, lower output in both the DM and CM, and a higher concentration of wealth.

These effects of trend inflation also show up in one-asset models of decentralized trading. In the no-heterogeneity model of Lagos and Wright (2005), for example, higher levels of trend inflation are associated with lower activity in the DM.[15] In Appendix D, I solve a one-asset model with persistent heterogeneity and study its equilibrium under different levels of trend inflation. In this model, higher levels of trend inflation lead money holdings to lose their value at a faster pace. This results in a reduced price of money and higher prices of both CM and DM goods. Thus, the liquidity risk associ-

[15]In Lagos and Wright (2005) the nominal interest rate is directly linked to trend inflation. Their one-asset model defines the implicit nominal interest rate on money as $(1+\pi)/\beta$. Hence, higher trend inflation implies higher nominal rates. In contrast, in my model, monetary policy can set a level of trend inflation while separately targeting a specific real interest rate, eliminating the negative first-order effects of higher inflation on decentralized trading.

Table 5: Long-Run Effects of Open Market Operations for Different Levels of Trend Inflation, Positive Government Expenditure

	$\overline{\mu} = 2.1\%$		$\overline{\mu} = 3.1\%$	
Nominal interest rate	4.9%	5.9%	4.9%	5.9%
Ratio bonds to money	2.0674	2.7630	1.3160	1.6265
Total output	0.4541	0.4549	0.4512	0.4519
Output in DM	0.1307	0.1311	0.1282	0.1288
Output in CM	0.3234	0.3238	0.3230	0.3231
Std. dev. money DM	0.0594	0.0457	0.0808	0.0680
Std. dev. money CM	0.7919	0.7908	0.7889	0.7922
Std. dev. bonds	1.4131	1.7633	1.0410	1.2142
Average DM price	0.4988	0.4965	0.5036	0.5042
Std. dev. DM price	0.0136	0.0117	0.0167	0.0152

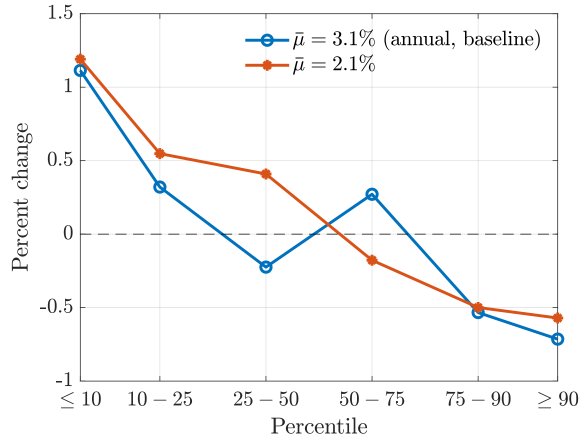
Notes: Stationary equilibria for two different levels of annual trend inflation, $\overline{\mu}$, and two different targeted annual nominal interest rates. This table presents the results for the case with $\tau = 0$ and $g > 0$.

ated with DM meetings increases, making it harder for agents to rebalance their money holdings, resulting in more unequal distribution of wealth across households. Moreover, this also generates lower quantities produced in DM meetings, while the distribution of prices of these meetings shifts to the right and becomes more dispersed.

Trend inflation also has an impact on the long-run redistributive effects of a change in monetary policy. Figure 8 shows how much CM consumption changes by wealth percentile groups. For most agents, consumption does not change significantly. However, having the increased supply of liquidity needed to implement a higher interest rate has a large positive impact on the poorest agents. As shown in Table 5, the supply of bonds needs to increase relatively more for the economy with lower trend inflation. This tends to benefit more the most liquidity-constrained agents, as the concentration of assets becomes more even and terms of trade move closer to the efficient allocation.[16]

[16] There is a special case when the real return on bonds is negative, i.e., when $\overline{\mu} > i$. In this case, agents have no incentives to have bonds in their portfolios and hence only demand money, as it still gives them the liquidity yield associated with decentralized trading. When this happens, the economy collapses to one in which money is the only asset and $K = A/M = 0$. See Appendix D for an economy for which this is the case.

Figure 8: Long-Run Distributional Effects on Centralized Market Consumption of an Open Market Operation and Trend Inflation, Positive Government Expenditure



Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates, under two different levels of trend inflation, $\overline{\mu}$. Changes are from an economy with a 4.9% annual nominal interest rate to one with a 5.9% rate. This figure presents the results for the case with $\tau = 0$ and $g > 0$.

5.3 Monetary Policy Changes at High and Low Levels of Trend Inflation

Changes in monetary policy that increase the relative supply of bonds also increase total output in a zero-inflation economy, as well as in economies with large levels of trend inflation. Table 6 presents the long-run effects of OMOs that target a 1 p.p. annual increase of the nominal interest rate when trend inflation is 0 percent and 10 percent, for the model with $\tau = 0$ and $g > 0$. For both cases, in the initial stationary equilibrium, the ratio of bonds to money is set as close as possible to the one in the baseline calibration, $K = 1.32$, and the nominal interest rate is then determined endogenously. As before, implementing a one-time permanent OMO that increases the nominal interest rate in 1 p.p. requires solving for the level of the supply of bonds that is consistent with that new rate.

As shown in the previous section, the more rapid the growth of nominal assets, the less attractive they are as they lose value faster. Thus, higher inflation levels are associated with lower output, higher DM prices, and an increased concentration of wealth. It also holds that permanent OMOs that target a higher interest rate require increasing the relative supply of nominal bonds. Such an increase is larger for lower levels of trend inflation. As reported in Table 6, the required increase is almost four times higher when

Table 6: Long-Run Effects of Open Market Operations for Low and High Levels of Trend Inflation, Positive Government Expenditure

	$\overline{\mu} = 0\%$		$\overline{\mu} = 10\%$	
Nominal interest rate (annual)	0.07%	1.07%	19.56%	20.71%
Ratio of bonds to money	1.2602	2.2592	1.3095	1.6123
Total output	0.4559	0.4568	0.4437	0.4443
Output in DM	0.1322	0.1329	0.1216	0.1222
Output in CM	0.3237	0.3239	0.3221	0.3221
Std. dev. of money in DM	0.1134	0.0590	0.0747	0.0644
Std. dev. of money in CM	0.6453	0.7802	0.7943	0.7934
Std. dev. of bonds	1.0363	1.5169	1.0407	1.2031
Average DM price	0.3992	0.4857	0.5245	0.5223
Std. dev. of DM price	0.0115	0.0127	0.0165	0.0149

Notes: For both levels of $\overline{\mu}$, the initial stationary equilibrium is computed while imposing that the ratio of bonds to money is as close as possible to the one in the baseline economy, $K = 1.32$. The second stationary equilibrium solves for the required level of K that implements an increase of 1 p.p. in the annual nominal interest rate. This table presents the results for the case with $\tau = 0$ and $g > 0$.

inflation is 0 percent than when it is 10 percent. Likewise, these OMOs increase output, reduce the concentration of wealth, and lower DM prices in the long run.

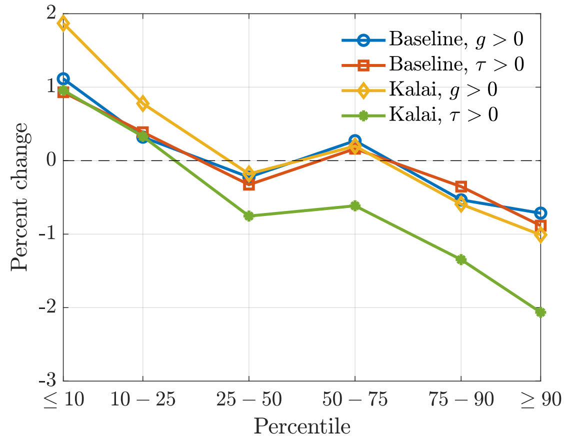
Importantly, these results are also related to the optimal conduct of monetary policy in the stationary equilibrium. Columns 1 and 3 of Table 6 and Column 3 of Table 5 show the stationary equilibrium for annual trend inflation equal to 0 percent, 10 percent, and 3.1 percent, respectively, when the relative supply of bonds is set to its calibrated value. As trend inflation approaches the one implied by the Friedman rule (see Section 7), nominal interest rates fall, while output increases.[17]

6 Robustness

I now turn to study an alternative version of the model where I explore the relevance of the bargaining protocol in influencing my results. I compare the effects of monetary policy changes in this counterfactual economy with the baseline version of the model. As before, this comparison considers both ways of closing the government balance.

[17] For the case of zero-trend inflation, the implied equilibrium nominal interest rate is slightly above zero. For other levels of K, having zero trend inflation does not necessarily imply a zero nominal interest rate. However, for any level of $K > 0$ that is consistent with the existence of a monetary equilibrium, there is a level of $\overline{\mu}$ such that the nominal rate is zero.

Figure 9: Long-Run Distributional Effects on Centralized Market Consumption of an Open Market Operation for Four Specifications of the Model



Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates, for three different model specifications (baseline, positive transfers, and Kalai bargaining). Changes are from an economy with a 4.9% annual nominal interest rate to one with a 5.9% rate.

To assess to what extent these results depend on the bargaining protocol, I solve an alternative version of the baseline model where terms of trade in DM meetings are determined by Kalai's proportional solution. As pointed out by Aruoba, Rocheteau and Waller (2007), the Kalai solution maintains strong monotonicity of agents' payoff as the bargaining set expands. With this bargaining protocol, terms of trade are determined according to the following problem:

$$\max_{q,d} \; S_b = u\left(q_m\right) + W\left(m_b - d_m + a_b + \tau; G, \theta\right) - W\left(m_b + a_b + \tau; G, \theta\right) \tag{18}$$

subject to a proportional total surplus-sharing condition

$$\theta_b S_s = \left(1 - \theta_b\right) S_b \tag{19}$$

and to $0 \leq d_m \leq m_b$, $q_m \geq 0$, where θ_b is the buyer's share of the total surplus, S_b is the buyer's surplus, and $S_s = -v\left(q_m\right) + W\left(m_s + d_m + a_s + \tau; G, \theta\right) - W\left(m_s + a_s + \tau; G, \theta\right)$ is the seller's surplus. Note that the proportional solution is equivalent to a take-it-or-leave-it offer when $\theta_b = 1$. Given that having $\theta_b < 1$ requires parametrizing an additional parameter, I re-calibrate the model to also match the ratio of liquid asset to annual output, in addition to the other moments targeted in the baseline model. In the 2007 SCF, this ratio is equal to 0.27, while the model reproduces a ratio of 0.31.

Table C3 depicts the change in DM consumption after an OMO that targets an increase in the nominal rate of 1 p.p. for the baseline case and Kalai bargaining case. I do this while taking into account both ways of closing the government balance. Note that, as discussed before, when $\tau > 0$, the lower transfers associated with higher interest payments on government debt drive poorer agents to self-insure more. With a more even distribution of assets, there is less redistribution from the top decile towards the lowest decile, as reflected in the consumption in the CM and DM (see Table 4).[18] However, with transfers in equilibrium being relatively small (0.55 percent of total output), the redistributional effects are not significantly different than in the baseline case, as shown in Figure 9.

As opposed to take-it-or-leave-it offers, sellers receive a positive share of the DM surplus with Kalai bargaining. This results in higher DM prices, so households have an increased and less dispersed demand for money entering the DM. Thus, there is a lower demand for bonds, which implies that the supply of relative bonds required to target a particular interest rate is less than in the baseline economy. Moreover, assets are more unevenly distributed, with a more significant increase in inequality in bond holdings.

Given this starting point, OMOs have the same qualitative implications but stronger redistributional effects than the baseline economy in the long run, as shown in Figure 9. The higher levels of liquidity that follow a one-time permanent OMO that targets an increase in the nominal interest rate increase the insurance availability for households at the bottom of the distribution. With Kalai bargaining, poorer households can increase their consumption levels by more in both DM and CM while reducing their labor supply to a larger extent. DM prices contract, as assets are more equally distributed and agents are better insured against liquidity shocks (see Tables C4-C5 in Appendix C).

These redistributional effects under Kalai bargaining are dampened in the lower end of the distribution when the government operates under positive lump-sum transfers. The fact that all agents receive some positive lump-sum transfer from the government reduces wealth inequality, and thus, meetings in the DM have terms of trade closer to the efficient level. However, the redistributional effects are amplified for asset-rich households. This group of households is affected by the lower dispersion of wealth and hence, the higher and less dispersed DM prices, resulting in higher losses after an OMO that targets a higher nominal interest rate.

[18]For an permanent OMO targeting a reduction in the interest rate, we have the opposite case (see Table C2).

7 Optimal Policy and the Welfare Cost of Inflation

A traditional question in the monetary economics literature has to do with the welfare cost of inflation. The welfare cost is measured in terms of consumption compensation. It considers how much consumption agents would sacrifice for not moving from a zero inflation equilibrium to one with positive inflation, usually 10 percent. Cooley and Hansen (1989) estimate this cost to be 0.4 percent of total output, while Lucas (2000) reports that it is slightly less than 1 percent, and Lagos and Wright (2005) suggest that it is 1.4 percent of consumption for their benchmark model. These papers, among many others, have traditionally relied on environments without persistent heterogeneity. In this type of setting, one of the major determinants of the cost of inflation is the inflation tax, i.e., the reduced incentive to hold cash and substitute away from activities that require it when inflation is higher.

In the model presented above, there are two conflicting forces. On the one hand, with higher inflation, agents holding more nominal assets experience a faster contraction in the real value of their portfolio. This means that the incidence of the inflation tax is not evenly distributed over the population when we have a nondegenerate distribution of money and nominal bonds. Moreover, part of the seigniorage money and new debt issuance are used to cover interest payments on government debt. Given that agents tend to hold just the money they need for transactional purposes, the concentration in bond holdings is key in determining which agents are receiving those interest payments. On the other hand, higher inflation rates are associated with larger lump-sum transfers. These mechanisms highlight the importance of distributional effects when measuring the cost of inflation.

In an environment similar to the one presented here, with persistent heterogeneity but with only one asset, Chiu and Molico (2011) estimate that the cost of inflation is even less than what Lucas (2000) documents because of the redistribution of resources towards the poorest agents. However, in the presence of only one asset, all seigniorage money is evenly distributed across agents. I follow a similar approach to theirs when computing the cost of inflation. First, note that the agents' average expected lifetime value when the steady-state inflation rate is μ is given by:

$$\mathfrak{U}(\mu) = \frac{1}{(1-\beta)}\left[\frac{\alpha}{2}\int_Z\int_Z \{u(q(z,\tilde{z};\mu)) - v(q(\tilde{z},z;\mu))\} F([dm\times da];\mu) F([d\tilde{m}\times d\tilde{a}];\mu) \right. \\ \left. + \int_Z U(c(z;\mu), h(z;\mu)) G([dm\times da];\mu)\right], \tag{20}$$

where $Z = \mathcal{M} \times \mathcal{A}$. Define the welfare cost of having an inflation of μ with respect to 0 as $\lambda(\mu) - 1$. This value captures how much of their consumption agents would be willing to sacrifice to not have an inflation rate equal to μ. We can compute $\lambda(\mu)$ by solving:

$$\mathfrak{U}(\mu) = \frac{1}{(1-\beta)} \left[\frac{\alpha}{2} \int_Z \int_Z \{u(q(z,\tilde{z};0)\lambda_0(\mu)) - v(q(\tilde{z},z;0))\} F([dm \times da];0) F([d\tilde{m} \times d\tilde{a}];0) \right.$$
$$\left. + \int_Z U(c(z;0)\lambda_0(\mu), h(z;0)) G([dm \times da];0) \right]. \quad (21)$$

Table 7 presents the computed values for $\lambda(\mu)$ for different values of the steady-state inflation rate, μ for the model with $\tau = 0$ and $g > 0$. For the case of 10 percent annual inflation, for example, the welfare quarterly cost is 1.14 percent, a value that lays in the middle between what Lagos and Wright (2005) and Chiu and Molico (2011) find. As discussed before, with higher inflation, agents are less interested in holding money balances, so the dispersion at the beginning of the DM falls. In contrast, there is an increased demand for bonds that pushes the dispersion in bond holdings upward and raises wealth inequality. As a result, DM prices increase in variance and in level.

However, a zero-inflation rate is not the welfare-maximizing policy. Table 7 also presents the welfare analysis for the case in which monetary policy implements the Friedman rule. Under the Friedman rule, the nominal interest rate is equal to 0. Because of the Fischer equation, under this policy it must be satisfied that $\phi_a = 1 + \pi$. This means that the Friedman rule is implemented when the gross growth rate of money and bonds is equal to the price of bonds. After solving for this level of μ, I find that under the Friedman rule there is trend deflation, with the annual money growth rate being -0.25 percent.

The Friedman rule maximizes welfare, with a marginal increase in welfare of 0.004 percent with respect to the zero-inflation case. It also implies lower and less dispersed DM prices, and less concentration in asset holdings. Importantly, the Friedman rule maximizes output, both in the CM and DM. Nevertheless, in contrast to Lagos and Wright (2005), implementing this rule does not imply eliminating the inefficiencies that arise in decentralized trading. These inefficiencies are always present at any inflation or nominal interest rate level. This happens because as long as two agents with different levels of wealth are matched in the DM, the quantity they trade is below the efficient level, i.e., $q(z,\tilde{z}) < q^*$ for $z \neq \tilde{z}$ as discussed in Footnote (8). Likewise, the presence of wealth effects in the CM, and the fact that not all agents rebalance their portfolios immediately after trading in the DM, guarantees that there will be persistent heterogeneity. Therefore, in this model, the Friedman rule is not optimal in the same sense as in Lagos and Wright (2005), where it implies that $q(z, \tilde{z}) = q^*$.

To understand these results, it is helpful to measure how much of the welfare changes can be attributed to changes in the distribution of agents, and to how much they adjust their behavior.[19] For low levels of inflation, changes in the distribution alone end up contributing to an increase in the welfare cost of inflation, as the dispersion in bond holdings and wealth increases, even if we maintain the same terms of trade and decision rules from the zero-inflation equilibrium. However, as inflation rises, the reduced concentration in money holdings dominates the increase in overall inequality and therefore reduces the cost of inflation. On the other hand, when focusing on the decision rules for the cases in which $\overline{\mu} > 0$, agents value less their nominal asset holdings with higher levels of trend inflation. This implies that they are willing to trade bilaterally at higher prices while wanting to accumulate more bonds that give them some positive real return. All these shifts in agents' behavior increase the welfare cost of inflation.

8 Concluding Remarks

In this paper, I study the long-run effects of changes in monetary policy while accounting for the interactions between aggregate variables and the distribution of assets across households. To do this, I develop a search-theoretic model of money where a consolidated fiscal and monetary authority can use OMOs to manage the public provision of liquidity. The model generates a nondegenerate distribution of money and bonds as agents do not adjust their asset holdings immediately after an idiosyncratic liquidity shock. Notably, the portfolio composition for agents along the distribution of wealth reproduces the one observed in the data.

My results show that a permanent increase in the relative supply of government bonds is associated with higher nominal interest rates and more economic activity. The increased supply of bonds reduces the concentration in asset holdings and total wealth while also expanding the chances for agents to self-insure against idiosyncratic liquidity shocks. Furthermore, as agents become more alike in terms of asset holdings, the inci-

[19] Rows 2 and 3 in Table 7 compute the cost of inflation that can be attributed to changes in only the distribution and in the decision rules alone. Importantly, these changes are not additive and cannot be interpreted as shares of the total welfare effects, as changes in both the distribution and decision rules may interact in compounding or dampening the effects of higher inflation rates. See Appendix B for a detailed discussion about how to isolate these two.

Table 7: Welfare Cost of Inflation Relative to 0% Trend Inflation, Positive Government Expenditure

Inflation rate (annual), $\overline{\mu}$	**-0.25%**	**0%**	**1%**	**2%**	**5%**	**10%**
Welfare cost (%)	-0.004	–	0.165	0.201	0.590	1.139
Change from distributions	-0.113	–	-0.232	-0.275	-0.257	-0.290
Change from decision rules	0.028	–	0.460	0.681	0.879	1.198
Average DM price	0.391	0.399	0.459	0.489	0.509	0.525
Std. dev. of DM price	0.011	0.011	0.015	0.016	0.017	0.017
Std. dev. of money in DM	0.123	0.113	0.103	0.084	0.078	0.075
Std. dev. of money in CM	0.635	0.645	0.731	0.772	0.791	0.794
Std. dev. of bonds	1.026	1.036	1.051	1.038	1.031	1.041

Notes: The welfare cost of inflation is measured as the percentage of consumption, in both the centralized and decentralized markets, that agents are willing to sacrifice to not have an inflation rate of $\overline{\mu}$, relative to a zero-inflation equilibrium. The welfare change explained by changes in the distributions measures changes in agents' average utility, while leaving the decision rules and DM terms of trade constant as in the equilibrium with inflation $\overline{\mu} = 0$. The welfare change explained by changes in decision rules does the same but keeps the distributions F and G as in the equilibrium with zero inflation. Each column is obtained by computing the stationary equilibrium for that particular level of the inflation rate $\overline{\mu}$, while imposing the ratio of bonds to money from the baseline economy, $K = 1.32$. The first column, in which there is trend deflation, denotes the Friedman rule. This table reports the results for the case with $g > 0$ and $\tau = 0$.

dence of frictions in bilateral trading shrinks so that agents, on average, trade at a level closer to the efficient one and at lower and less dispersed prices. As a result, total output increases, and the poorest and more liquidity-constrained agents gain the most, as some resources get redistributed out of the wealthiest agents in the economy.

I also find asymmetries in the response of the economy to changes in monetary policy. When targeting interest rate hikes, the size of the required OMO is larger than when implementing an equivalent interest rate contraction. Moreover, this asymmetry also affects total output. By cutting the public provision of liquidity, agents become less able to self-insure. This, in turn, exacerbates the negative consequences of liquidity shocks while increasing the probability of having DM meetings with trade further away from the efficient level. Such an effect is stronger when the supply of liquidity falls, as more agents are pushed to be liquidity constrained.

Finally, the presence of heterogeneity across agents and the existence of a fully-fledged distribution of prices suggest that an OMO has enough room to generate short-run non-neutralities. In particular, since agents cannot rebalance their portfolio instantaneously after a liquidity shock, the distribution of assets and the distribution of prices would slowly respond to the increased liquidity. This extension is worth considering in future research.

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Online Appendix for The Long-Run Redistributive Effects of Monetary Policy

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A Solution Method

My method solves iteratively for the agents' decision rules, the terms of trade in decentralized trading, the level of lump-sum transfers consistent with the government balance, and the prices that clear the markets of bonds and money. We start by solving the agents' value functions in (9) and (10). This requires updating $V(m, a; F, \theta)$ and $W(x; G, \theta)$ one after the other. Doing this for W in the decentralized market (DM) is standard, as long as we have how to compute the continuation value V. However, when computing the value function in the DM, we need to know the entire distribution $F(m, a) : \mathcal{M} \times \mathcal{A} \to \mathbb{R}$ and the terms of trade for each possible meeting between agents with (m_b, a_b) and (m_s, a_s), where $(m_i, a_i) \in \mathcal{M} \times \mathcal{A}$ for $i = b, s$. Therefore, we must start not only with some guess for W, as in any value function iteration approach, but also with guesses for $F(m, a)$ and $\langle Q, D \rangle = \langle q(m_b, a_b, m_s, a_s; F, \theta), d(m_b, a_b, m_s, a_s; F, \theta) \rangle$. To do this, we discretize our state space $\mathcal{M} \times \mathcal{A}$, so that $F(m, a)$ and $\langle Q, D \rangle$, as well as $V(m, a; F, \theta)$, are defined over a finite number of points. Similarly, when evaluating $W(x; G, \theta) : \mathcal{X} \to \mathbb{R}$, we need to discretize $\mathcal{X}$. Given this, we can solve the stationary equilibrium of this economy as follows:

1. Solve for V and W given some $F(m, a)$, some terms of trade $\langle Q, D \rangle$, a vector of prices (ϕ_m, ϕ_a), and some guess for the transfers, τ. As a byproduct of this step, we should have the decision rules for $m' = g_m(x; G, \theta)$ and $a' = g_a(x; G, \theta)$ in the centralized market (CM).

   There are several valid approaches to this. In particular, I use piece-wise cubic splines when evaluating continuation values that depend on W, solve the non-linear labor-leisure condition using the bisection method, and employ Howard's improvement algorithm to speed up the solution of the CM problem in (10)-(12).

2. Given $F(m,a)$ and the current values in $\langle Q, D\rangle$, compute $G(m,a)$. Likewise, using $G(m,a)$ and the decision rules for m' and a', compute $F'(m,a)$. Repeatedly update F and G until these distributions converge.

3. Verify if markets for money and bonds are clearing, i.e., check if Equations (5) and (6) hold. If not, adjust the prices (ϕ_m, ϕ_a) and return to Step 1. Repeat this process until clearing both markets. Of course, when returning to Step 1, use the most recently computed value for F.
   When adjusting prices, an alternative is to use the excess demand functions of bonds and money. I do so using a partial updating approach.

4. Government:

   (a) If solving the baseline economy with $\tau = 0$ and $g > 0$, use the government balance in Equation (2) to determine g.

   (b) If solving the case with $\tau > 0$, check if the government balance in Equation (2) is satisfied. In case it is not, adjust the tax rate in Equation (3) using the most recent value for ϕ_a and go back to Step 1.

5. Finally, update the terms of trade $\langle Q, D\rangle$. The key input here is the value function W that is required to compute the continuation values in the problem in (7)-(8). As in Step 1, I use piece-wise cubic splines on W. Check if the newly calculated terms of trade are close enough to their previous values. If not, return to Step 1. If so, we are done.

B Decomposing Welfare Changes

Welfare changes are the result of changes in the distribution, as well as changes in the decision rules and DM terms of trade. The following explain how to look at each one of these independently. First, note that the average lifetime utility in Eq. (20) can be rewritten as:

$$\mathfrak{U}(\mu) = \frac{1}{(1-\beta)}\left[\frac{\alpha}{2}\int_Z\int_Z \{\Lambda(z,\tilde{z},q(z,\tilde{z};\mu), d(z,\tilde{z};\mu), c(z;\mu), h(z;\mu)\}\right.$$
$$\left. \times F([dm \times da];\mu) F([d\tilde{m} \times d\tilde{a}];\mu)\right], \tag{B.1}$$

where the function Λ captures the fact that the average lifetime utility is computed using the terms of trade $\{q(\cdot;\mu), d(\cdot;\mu)\}$ and decision rules $\{c(\cdot;\mu), h(\cdot;\mu)\}$ when the inflation is μ, and the distribution $F(\cdot;\mu)$ when the inflation rate is μ as well.

We can use this same function to compute the average lifetime utility of having the terms of trade and decision rules implied by $\mu = 0$, while using the $\mu > 0$ distribution:

$$\mathfrak{U}_F(\mu) = \frac{1}{(1-\beta)}\left[\frac{\alpha}{2}\int_Z\int_Z \{\Lambda(z,\tilde{z},q(z,\tilde{z};0), d(z,\tilde{z};0), c(z;0), h(z;0)\} \times F([dm \times da];\mu)\, F([d\tilde{m} \times d\tilde{a}];\mu)\right]. \tag{B.2}$$

This yields row 2 of Table 7, in which we focus on the effect of changes distribution alone. In contrast, if we compute the average lifetime utility resulting from the terms of trade and decision rules implied by $\mu > 0$, but applying the $\mu = 0$ distribution:

$$\mathfrak{U}_{DR}(\mu) = \frac{1}{(1-\beta)}\left[\frac{\alpha}{2}\int_Z\int_Z \{\Lambda(z,\tilde{z},q(z,\tilde{z};\mu), d(z,\tilde{z};\mu), c(z;\mu), h(z;\mu)\} \times F([dm \times da];0)\, F([d\tilde{m} \times d\tilde{a}];0)\right]. \tag{B.3}$$

we obtain row 3 of Table 7. This allows us to focus on the role of changes in decision rules and terms of trade by themselves.

C Additional Figures and Tables

Table C1: Elasticity of Model Moments with Respect to Parameters

Parameter	α	B	ν	χ	κ
Ratio of bonds to money	2.45	0.00	0.14	1.34	2.07
Total output	0.19	0.00	-0.30	0.16	-3.25
Output in D	0.68	0.00	-1.04	0.19	-2.55
Output in CM	0.00	0.00	-0.01	0.16	-3.51
Std. dev. money DM	-1.80	0.00	-8.05	-2.78	-30.56
Std. dev. money CM	0.34	0.00	-1.06	-0.10	-2.72
Std. dev. of bonds	2.04	0.00	0.86	1.60	6.46
Average DM price	0.04	0.15	0.75	-0.07	-2.99
Std. dev. DM price	-0.31	0.15	-1.41	-0.58	-12.82
Velocity	0.19	0.00	-0.26	-0.10	-3.17

Notes: Elasticities of model moments (rows) with respect to parameters (columns) around the calibrated parameters for the baseline stationary equilibrium. α is the probability of being matched in the DM, B and ν are the scale and curvature parameters of the cost of working in the DM, χ is the inverse of the Frisch elasticity of labor supply in the CM, and κ is the scale parameter for the disutility of labor in the CM.

Table C2: Long-Run Distributional Effects of an Open Market Operation from $i = 4.9\%$ to $i = 3.9\%$

Percentile	≤ 10	**10-25**	**25-50**	**50-75**	**75-90**	≥ 90
Positive gov. expenditure						
Consumption (CM)	-1.02	-0.27	-0.02	0.30	0.22	0.19
Labor	2.21	0.61	0.05	-0.67	-0.47	-0.39
Money holdings	-2.02	0.66	0.01	-0.31	2.06	-0.77
Bond holdings	–	-45.15	-20.02	-14.96	-13.76	-10.76
Consumption (DM)	-0.74	0.08	-0.24	0.48	-0.03	-0.11
Positive transfers						
Consumption (CM)	-0.7	-0.1	-0.3	0.3	0.5	0.6
Labor	1.5	0.3	0.8	-0.7	-1.1	-1.3
Money holdings	-2.7	-2.3	0.0	3.1	0.7	-0.5
Bond holdings	–	-29.6	-18.4	-17.6	-14.7	-12.9
Consumption (DM)	-1.0	-0.6	-0.3	1.6	-0.3	0.2

Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates for both ways of closing the model. Changes are with respect to the baseline economy with an annual 4.9% nominal interest rate. In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$. Agents in the lowest decile have zero bond holdings in both stationary equilibria.

Table C3: Percent Changes in Stationary Equilibrium for Given Changes in the Nominal Interest Rate. Case of Kalai Bargaining

	Positive gov. expenditure		**Positive transfers**	
Nominal interest rate	-1p.p.	$+1$p.p.	-1p.p.	$+1$p.p.
Ratio bonds to money	-13.69	24.32	-16.37	24.60
Total output	-0.31	0.20	-0.13	0.20
Output in DM	-0.64	0.46	-0.46	0.73
Output in CM	-0.09	0.03	0.09	-0.15
Std. dev. money DM	14.37	-21.04	17.44	-19.63
Std. dev. money CM	-2.21	0.65	-3.28	0.38
Std. dev. bonds	-9.28	16.70	-9.94	17.40
Average DM price	-1.75	-0.25	-3.12	-1.35
Std. dev. DM price	16.78	14.99	-10.05	0.64
Transfers	–	–	20.83	-23.61

Notes: Percentage change in stationary equilibria moments for two different targeted nominal interest rates with respect to the baseline calibration and for both ways of closing the model. In each case, the baseline scenario targets an annual 4.9% rate, while the two additional scenarios target a 1 p.p. change relative to the baseline rate (3.9% and 5.9%). Terms of trade in DM meetings are determined by Kalai bargaining. In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$.

Table C4: Long-Run Distributional Effects of an Open Market Operation from $i = 4.9\%$ to $i = 5.9\%$. Case of Kalai Bargaining

Percentile	≤ 10	**10-25**	**25-50**	**50-75**	**75-90**	≥ 90
Positive gov. expenditure						
Consumption (CM)	1.87	0.78	-0.18	0.20	-0.59	-1.01
Labor	-3.76	-1.56	0.34	-0.41	1.18	2.04
Money holdings	3.20	3.23	-0.31	0.77	-3.01	-3.70
Bond holdings	–	47.60	36.62	22.99	21.94	19.22
Consumption (DM)	3.24	2.31	0.67	0.20	-1.03	-0.89
Positive transfers						
Consumption (CM)	1.66	0.38	0.18	-0.20	-1.01	-1.32
Labor	-3.40	-0.74	-0.34	0.37	2.01	2.76
Money holdings	3.04	3.11	-0.02	0.16	-2.07	-3.19
Bond holdings	–	49.95	32.01	24.01	22.71	19.51
Consumption (DM)	2.78	2.27	0.85	0.33	-0.62	-0.55

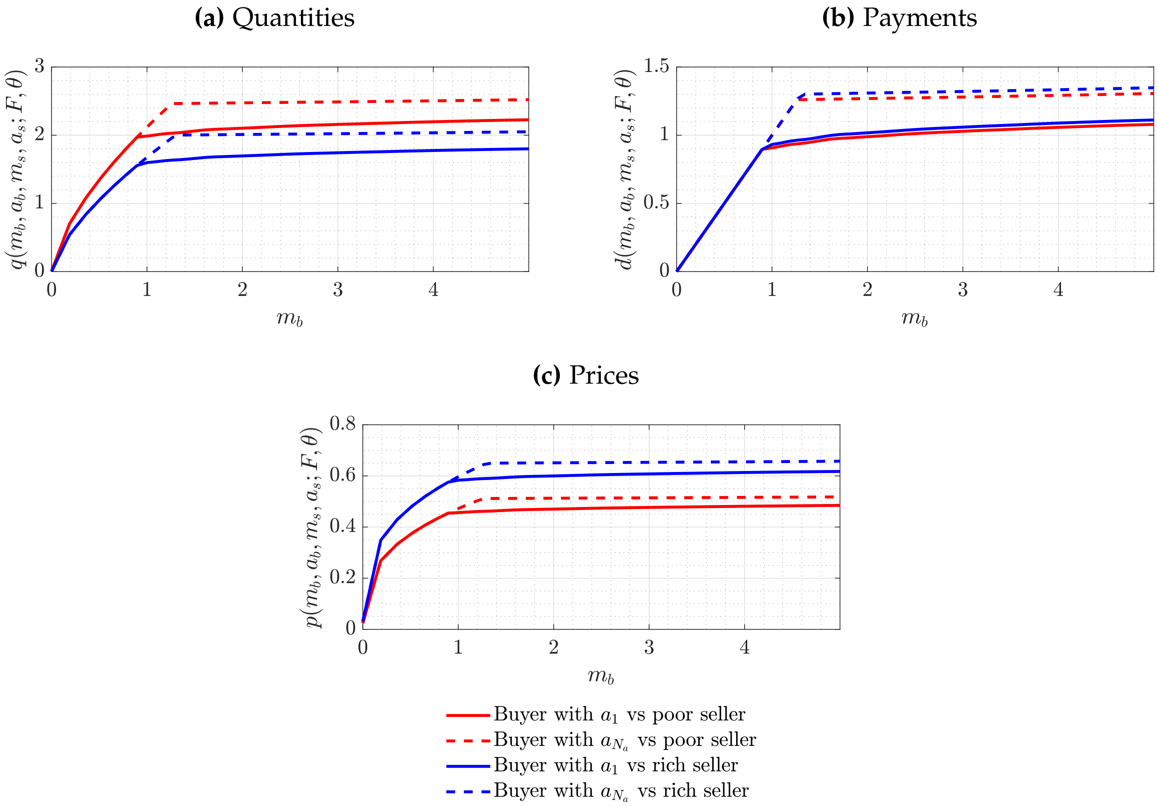
Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates or both ways of closing the model. Changes are with respect to the baseline economy with an annual 4.9% nominal interest rate. Agents in the lowest decile have zero bond holdings in both stationary equilibria. Terms of trade in DM meetings are determined by Kalai bargaining. In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$. Agents in the lowest decile have zero bond holdings in both stationary equilibria.

Table C5: Long-Run Distributional Effects of an Open Market Operation from $i = 4.9\%$ to $i = 3.9\%$. Case of Kalai Bargaining

Percentile	≤ 10	**10-25**	**25-50**	**50-75**	**75-90**	≥ 90
Positive gov. expenditure						
Consumption (CM)	-0.75	-0.67	-0.10	-0.16	0.32	0.80
Labor	1.68	1.32	0.25	0.30	-0.59	-1.58
Money holdings	-5.17	-1.28	0.53	0.58	0.97	-0.59
Bond holdings	–	-28.97	-18.38	-12.92	-12.22	-10.28
Consumption (DM)	-1.80	0.58	0.06	0.95	0.40	0.76
Positive transfers						
Consumption (CM)	-1.60	-0.07	0.10	0.07	0.27	0.73
Labor	3.11	0.18	-0.21	-0.08	-0.56	-1.36
Money holdings	-6.33	-1.70	2.51	-0.15	0.93	-0.58
Bond holdings	–	-33.48	-26.15	-16.63	-14.15	-11.37
Consumption (DM)	-2.10	0.79	0.14	1.06	0.68	1.17

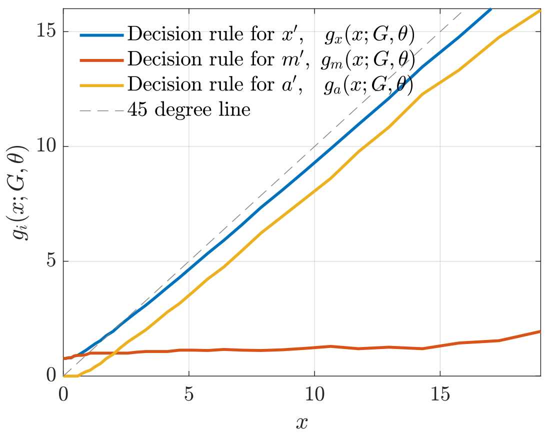
Notes: Percent changes by percentile groups across stationary equilibria with different targeted nominal interest rates or both ways of closing the model. Changes are with respect to the baseline economy with an annual 4.9% nominal interest rate. Agents in the lowest decile have zero bond holdings in both stationary equilibria. Terms of trade in DM meetings are determined by Kalai bargaining. In the positive government expenditure case, $g > 0$ and $\tau = 0$, while in the positive lump-sum transfer case, $g = 0$ and $\tau > 0$. Agents in the lowest decile have zero bond holdings in both stationary equilibria.

Figure C1: More on Terms of Trade: Quantities, Monetary Payments, and Prices as Function of Money from Buyer, m_b



Notes: Terms of trade as a function of buyer's money holdings. This includes the outcomes for combinations of meetings between either bond-poor ($a_b = a_1$) or bond-rich ($a_b = a_N$) buyers, and either asset-poor ($z_s = (m_1, a_1)$) or asset-rich ($z_s = (m_{N_m}, a_{N_a})$) sellers. In the numerical solution, m_1 and a_1 are the lowest possible value for money and bond holdings, while m_{N_m} and a_{N_a} are the highest. As a reference, the efficient level of trading for the baseline calibration is $q^* = 2.24$.

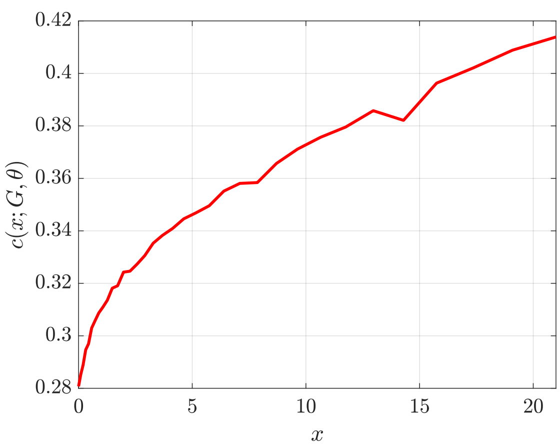
Figure C2: Decision Rules for Assets



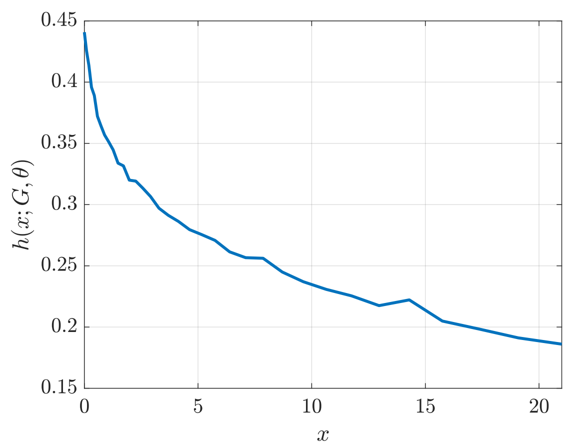
Notes: Decision rules for money, $m' = g_m(x; G, \theta)$; bonds, $a' = g_a(x; G, \theta)$; and the implied total wealth, $x' = m' + a' = g_x(x; G; \theta)$, as a function of wealth, x, in the centralized market.

Figure C3: Decision Rules for Consumption and Labor in the CM

(a) Consumption in the CM



(b) Labor in the CM



Notes: Decision rules for consumption, $c = c(x; G, \theta)$, and labor, $h' = h(x; G, \theta)$, as a function of wealth, x, in the centralized market.

D Persistent, Nondegenerate Distribution with Only Money

Below I present a version of the model where money is the only asset and, hence, monetary policy does not operate through OMOs but via lump-sum transfers to agents. This model economy resembles the one in Lagos and Wright (2005), but with non-quasi-linear preferences. This simplified model illustrates how much heterogeneity this environment can generate as it moves away from having a quasi-linear utility function in the CM and we add more curvature to the labor supply. It is also helpful to explain the numerical solution method in a simpler manner.

As in the full model, each period is divided into decentralized and centralized markets operating sequentially. Matching in the DM occurs as in the full model, with agents trading using only money and terms of trade being determined by a "take it or leave it" offer of the buyer to the seller. The agents' period utility function is as in the full model, with U not restricted to be linear in h. The production technologies in both the decentralized and centralized markets operate as in the full model.

The monetary authority controls the supply of the only available asset to agents: fiat money, M. The supply of money grows at the rate μ. At the beginning of every CM, the monetary authority transfers μM to every agent in the economy before the CM begins. This implies that the stock of aggregate nominal money evolves according to:

$$M' = (1+\mu)\, M, \tag{D.1}$$

where M' is the nominal money supply at the middle of the current period, and therefore, at the beginning of next period. Individual nominal money holdings are normalized with respect to the beginning of the period money supply, M. This means that if an agent holds $\hat{m}$ units of nominal money, then they have $m = \hat{m}/M$ units of relative money holdings. Note that money injections μM are equivalent to μ units of relative money.

Following a similar notation as before, the distribution of agents over m are summarized by the probability measures $F(m)$ and $G(m)$ for the DM and CM, respectively. These distributions evolve according to $G = \Gamma_G(F, \mu)$ and $F' = \Gamma_F(G, \mu)$. Given this, and the policy implemented by the monetary authority, in equilibrium we have:

$$\int_{\mathcal{M}} m dF(m) = 1 \tag{D.2}$$

$$\int_{\mathcal{M}} m dG(m) = 1 + \mu. \tag{D.3}$$

Let $V(m;F,\mu)$ and $W(m;G,\mu)$ be the value functions at the beginning of the DM and CM, respectively. As noted before, without loss of generality, we assume that there are not double-coincidence meetings. In single-coincidence meetings, a buyer and a seller with relative money holdings m_b and m_s are matched. The buyer makes a "take it or leave it" offer to the seller: the buyer offers to buy q of the differentiated good at the price d. The terms of such an offer are determined according to the following problem:

$$\max_{q,d} \; u(q) + W(m_b + \mu - d; G, \mu) \tag{D.4}$$

subject to the seller's participation constraint

$$-v(q) + W(m_s + \mu + d; G, \mu) \geq W(m_s + \mu; G, \mu) \tag{D.5}$$

to the law of motion of the distribution, $G(m) = \Gamma_G(F(m), \mu)$, and to $0 \leq d \leq m_b$, $q \geq 0$. Note that the continuation values of buyers and sellers take into account their money holdings when exiting the current DM and the transfer they will receive at the beginning of the next CM. For each meeting (m_b, m_s), and given an aggregate state $\{F(m), \mu\}$, we have that the terms of trade $q(m_b, m_s; F, \mu)$ and $d(m_b, m_s; F, \mu)$ solve the problem stated above.

In this context, the expected lifetime utility at the beginning of the DM of an agent with money holdings m, i.e., before knowing if they are matched or not, and before knowing their role in an eventual match, is given by the following functional equation:

$$\begin{aligned} V(m;F,\mu) = &\frac{\alpha}{2}\int_{\mathcal{M}} \{\quad u(q(m,m_s;F,\mu)) + W(m+\mu-d(m,m_s;F,\mu);G,\mu)\}\, dF(m_s) \\ &+ \frac{\alpha}{2}\int_{\mathcal{M}} \{-v(q(m_b,m;F,\mu)) + W(m+\mu+d(m_b,m;F,\mu);G,\mu)\}\, dF(m_b) \\ &+ (1-\alpha)\, W(m+\mu;G,\mu). \end{aligned} \tag{D.6}$$

Let $\tilde{m}$ denote relative money holdings at the beginning of the CM. Money holdings at the beginning of the CM are those at the end of the previous DM, plus lump-sum transfers of money. For an agent with relative money holdings $\tilde{m}$, the lifetime value of their utility at the beginning of the CM is given by:

$$W(\tilde{m};G,\mu) = \max_{c,h,m'} \left\{ U(c,h) + \beta V\left(m';F',\mu\right) \right\} \tag{D.7}$$

subject to the budget constraint

$$c = h + \phi(G, \mu)\left[\tilde{m} - (1+\mu)\, m'\right] \tag{D.8}$$

and to the mapping of $G(m)$ into $F'(m)$

$$F'(m) = \Gamma_F(G(m), \mu). \tag{D.9}$$

For the utility and cost functions in the DM, as in the full model, I follow Lagos and Wright (2005). However, for the CM I drop the quasi-linearity assumption. In particular, the utility function in the CM is concave in both consumption and leisure and given by:

$$U(c, h) = \log c - \kappa \frac{h^{1+\chi}}{1+\chi}, \tag{D.10}$$

where χ is the inverse of the Frisch elasticity of labor supply and $\kappa > 0$ a scale parameter. Note that Eq. (D.10) nests the quasi-linear utility function in Lagos and Wright (2005) when $\chi = 0$ and $\kappa = 1$.

Denoting by π the growth rate of prices, we can then define the implicit nominal interest rate on money as in Lagos and Wright (2005):

$$1 + i = (1+r)(1+\pi) = \frac{1}{\beta}(1+\pi) \tag{D.11}$$

The model is parametrized as closely as possible to Lagos and Wright (2005) for comparison purposes. One model period is equivalent to one year. The discount factor is set to generate a real interest rate of 4 percent, and the money growth rate is consistent with an inflation rate, in the stationary equilibrium, of 2 percent. The probability of being matched in the DM, α, is set to 1 to minimize the role of the search frictions. The inverse of the Frisch elasticity is chosen to be 0.5. I select B and κ (scale parameters), as well as ν (the curvature parameter that pins down the cost of producing in the DM) to reproduce the following three targets: (1) the size of the CM to be 10 percent of total output, (2) a velocity of money of 2, and (3) an average markup in DM meetings of 30 percent (Faig and Jerez, 2005). Finally, the parameter that governs the curvature of the utility function in the DM, η, is set close to 1, so that the utility function is close to being logarithmic.

D.1 Stationary Equilibrium

In a stationary equilibrium, the distribution over relative money holdings remains constant, i.e., $F = \Gamma_F(\Gamma_G(F,\mu),\mu)$. This, in turn, implies $\phi = \phi'$ and requires the growth rate of prices, π, to be equal to the growth rate of money, μ. In contrast to models like Shi (1997) or Lagos and Wright (2005), the distribution of money holdings is not necessarily degenerate across periods.

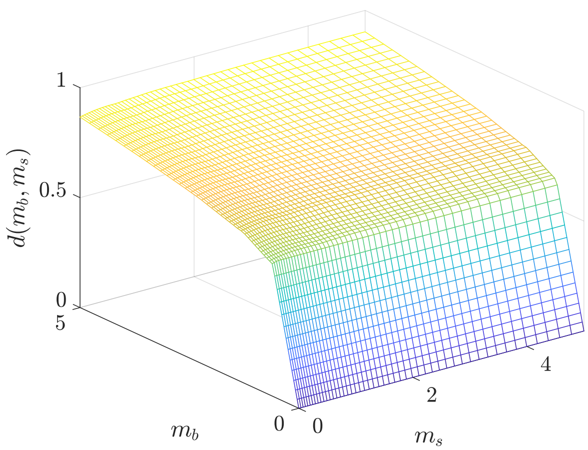
Figure D4 shows the stationary terms of trade $d(m_b, m_s; F, \mu)$ and $q(m_b, m_s; F, \mu)$. As opposed to models where preferences in the CM are linear in labor, here, as in the full model, terms of trade depend not only on the buyer's money holdings but also on the seller's. For a fixed level of m_s, larger values of m_b imply that the buyer can obtain more of the differentiated good but at a higher total cost. On the other hand, keeping fixed m_b, a seller with larger money balances requires a higher payment for lower quantities of the good he produces. This is because of the seller's continuation value (captured by the function W), which is now concave in m. Consequently, as Figure D4(c) shows, the price per unit of good exchanged, $p_d(m_b, m_s) = d(m_b, m_s)/q(m_b, m_s)$, mainly responds to the seller's wealth.

The chance of having different types of meetings with heterogeneity in both q and d, plus the fact that preferences in the CM are not quasi-linear, generates a nondegenerate distribution of money holding, as shown in Figure D5(a). In that figure, we have the distributions $F(m)$ and $G(m)$. The distribution entering the CM, unsurprisingly, is more dispersed than the distribution at the beginning of the DM, reflecting the heterogeneity-inducing effect of decentralized trading. We can also observe that, after trading in the DM, some agents end up with very little money holdings (the money injections prevent them from reaching $m = 0$). This is the result of a combination of sequences of meetings with sellers that make the buyer deplete her money holdings. However, even for this "unlucky" type of agent, the CM acts as an insurance market that lets her replenish, at least partially, her liquidity for the next round of decentralized trading. Note that some agents reaching the lower end of the distribution may experience similar matches in the future as they move away from $m = 0$. This explains the spikes in $F(m)$ and why they get diluted for higher values of money holdings.

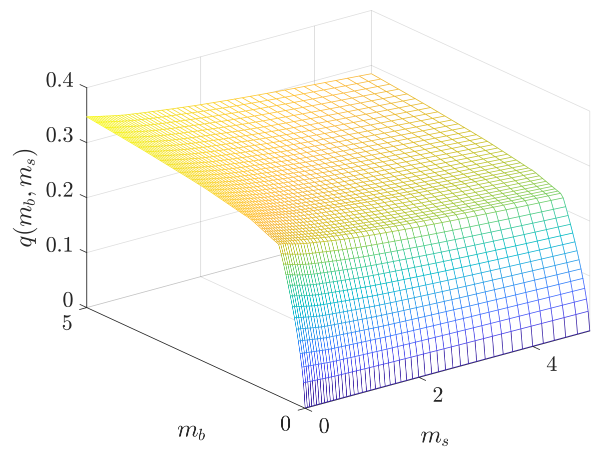
Figure D5(b), shows the observed distribution of prices. This distribution comes from taking into account the likelihood of all possible meetings implied by the distribution $F(m)$ and the prices $p_d(m_b, m_s)$ at each one of them. In this context, the model is able

Figure D4: Terms of Trade in the Decentralized Market

(a) Payments



(b) Quantities



(c) Price per unit

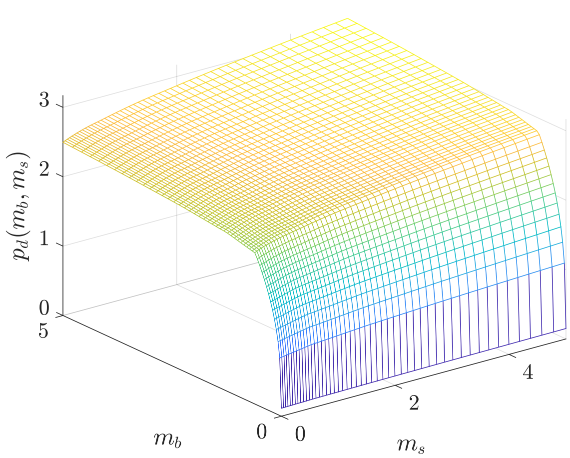
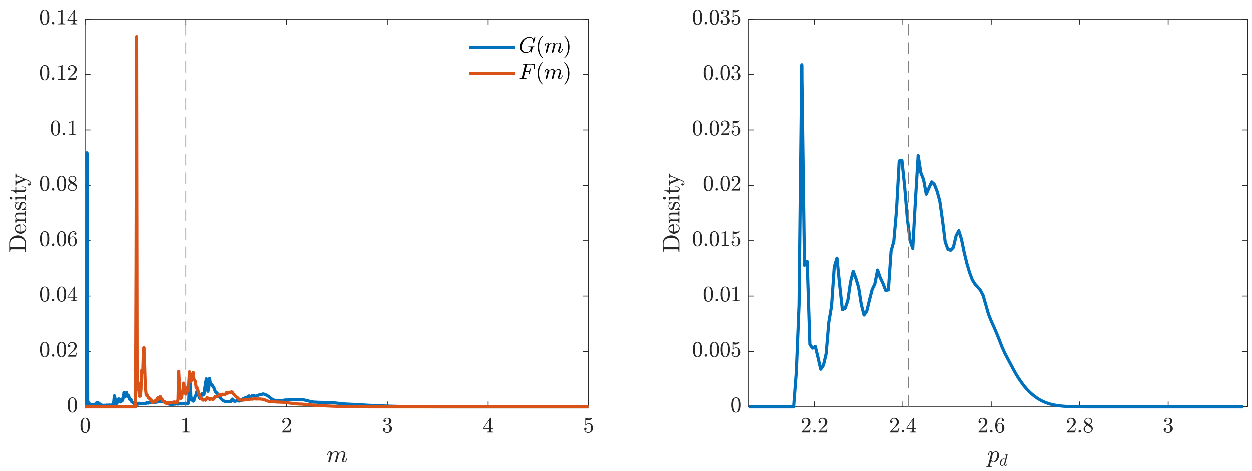


Figure D5: Distributions at the Stationary Equilibrium

(a) Distribution of Money Holdings **(b)** Distribution of Prices



to produce a nondegenerate distribution of prices that reflects the differences in assets between different agents in this economy.

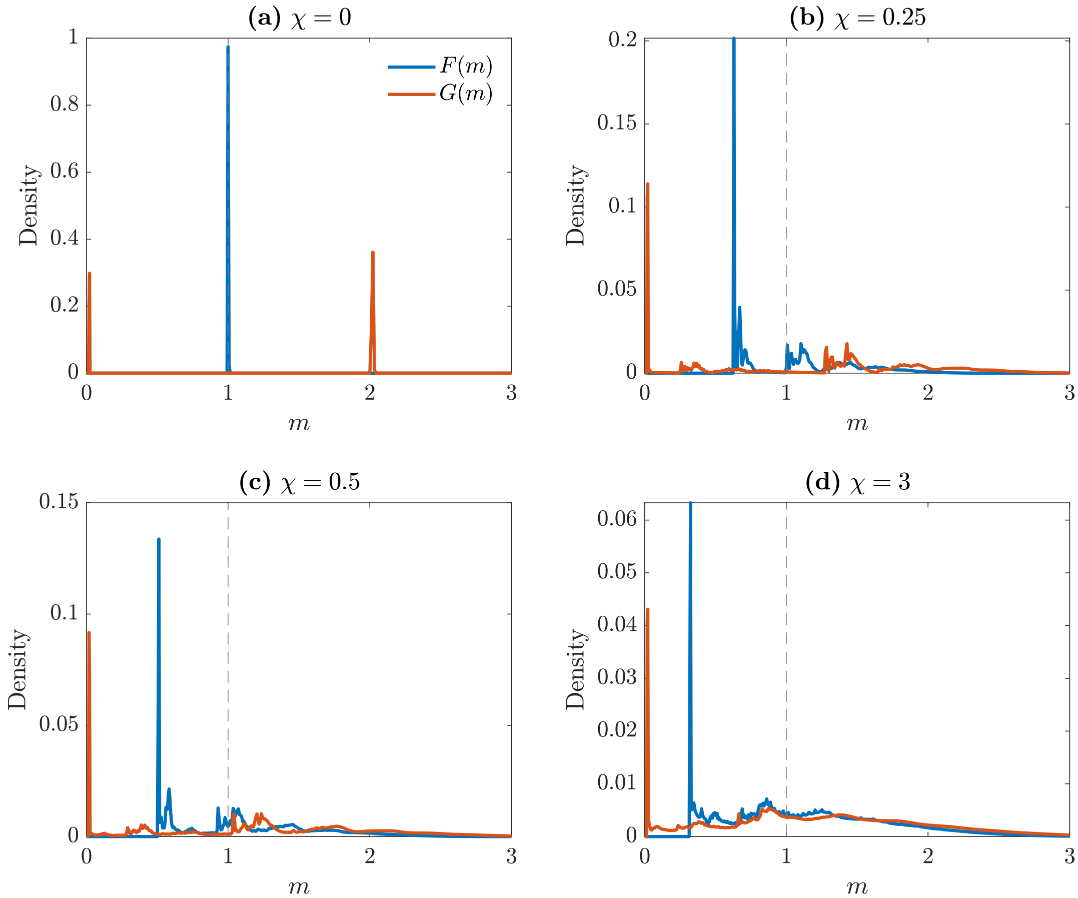
D.2 The Role of the Elasticity of Labor Supply

The elasticity of labor supply is crucial in determining the extent to which agents can reshuffle their money holdings after trading in the DM. The higher this elasticity, the easier for agents to offset their previous idiosyncratic trading histories. In Lagos and Wright (2005), labor supply is infinitely elastic, so agents are able to completely undo in the CM whatever happened in their previous DM meeting. In this model, having a perfectly elastic labor supply is equivalent to $\chi = 0$.

Figure D6 shows the stationary distributions of money at the beginning of the DM and CM for different values of χ, the inverse of the Frisch elasticity of labor. When $\chi = 0$, we obtain a degenerate distribution of money in the DM (blue line in panel (a) of Figure D6): all agents are holding the same amount of relative units of money. As these agents trade in decentralized meetings, buyers deplete their money holdings, while sellers end up with about double what they initially had (red line).[20] However, the fact that, for this case, labor is perfectly elastic allows agents to reshuffle back their money holdings completely. The decision rules for labor and money holdings in Figure D7(a) and Figure

[20]When $\chi = 0$, the CM distribution is split equally between two points, each one with mass 0.5. In Figure D6, this does not appear to happen because of a small numerical error. These two points are not necessarily in the grid $\mathcal{M}$, so that the mass of agents gets distributed between their adjacent points. In any case, adding the masses at those adjacent points gives exactly 0.5 for each one.

Figure D6: Distributions of Money Balances at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor

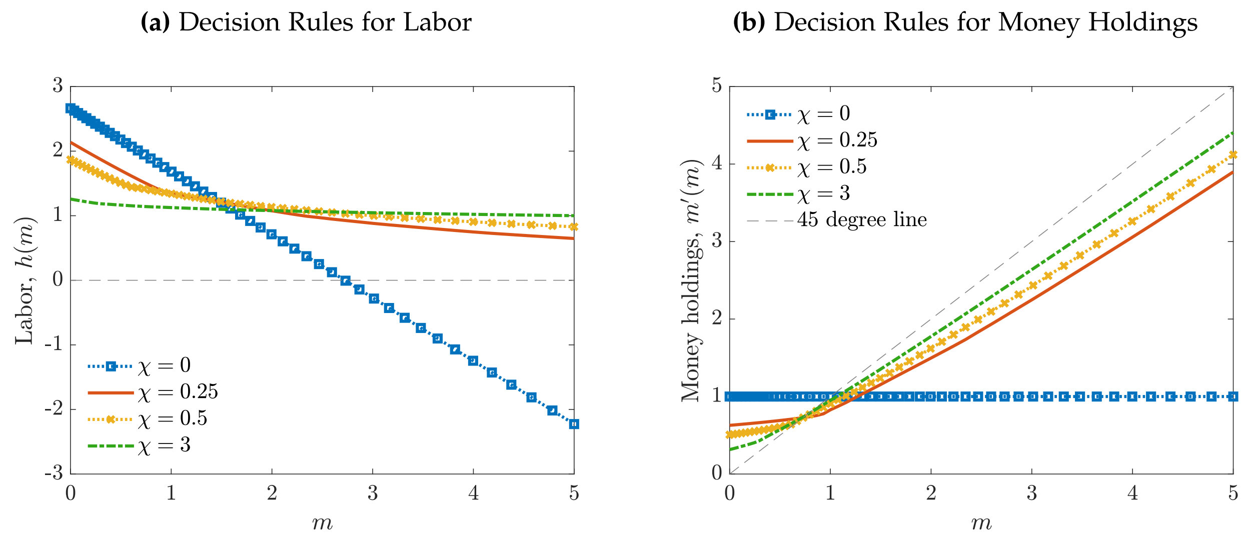


Notes: Stationary distributions of money holdings at the beginning of the decentralized market, $F(m)$, and the beginning of the centralized market, $G(m)$, for different values of the Frisch elasticity of labor supply, $1/\chi$.

D7(b) show that labor supply changes linearly with the individual state, m, so that agents can exactly exit the period with 1 unit of relative money. This is a consequence of the absence of wealth effects under quasi-linear preferences.

When the Frisch elasticity of labor supply ($1/\chi$) becomes finite, the inability of agents to offset the effects of their previous trading history results in a nondegenerate distribution of money in the DM. In fact, as the elasticity of labor supply decreases, we obtain distributions of money holdings with less concentration.

Figure D7: Decision Rules in the Centralized Market at the Stationary Equilibrium for Different Levels of the Frisch Elasticity of Labor



D.3 The Role of Trend Inflation

Trend inflation also plays an important role in determining equilibrium outcomes. The growth rate of money directly determines trend inflation. To illustrate the effects of higher levels of trend inflation, I solve multiple stationary equilibria under different levels for it. In particular, I consider $\mu = \{0.01, 0.02, 0.10, 0.20\}$, where the benchmark parametrization discussed earlier for the one-asset model considers a 2 percent trend inflation rate, i.e., $\mu = 0.02$.

Table D6 presents the levels of different variables at the stationary equilibrium under different values of μ. Figure D8, shows how some variables of interest change relative to the benchmark parametrization. With higher levels of trend inflation, money loses value more rapidly, resulting in higher DM prices, a higher concentration of money, and, consequently, lower DM output. Furthermore, higher levels of trend inflation are associated with lower prices of money (in terms of the CM good). This, in turn, implies that the nominal price of CM goods increases with μ.

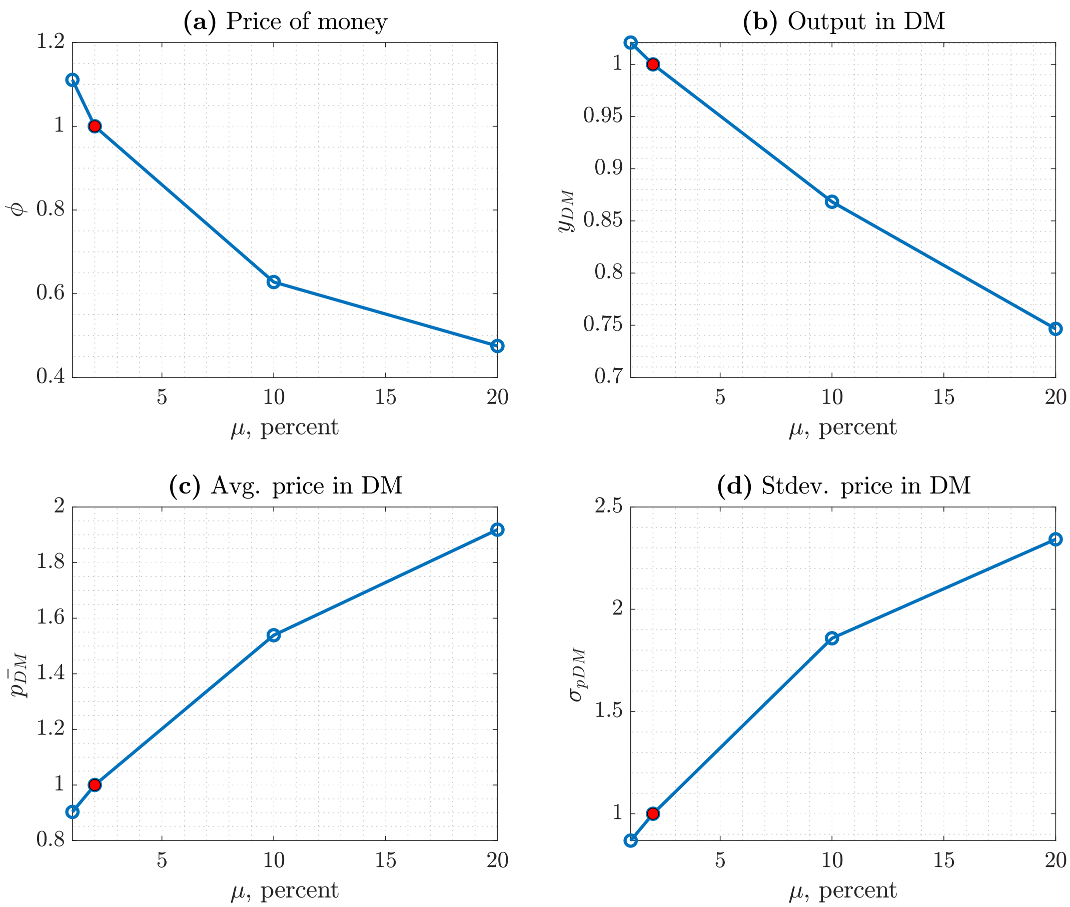
Consequently, agents tend to reduce total consumption (equal to output in this economy) and their demand for money balances. Given that the stationary money supply is normalized to 1, lower levels of economic activity are translated into a higher velocity of money. This is consistent with what has been previously reported by other papers in the literature of search-theoretic models of money (see for example Lagos and Wright, 2005; Chiu and Molico, 2011). Furthermore, consumption in the decentralized market

Table D6: Stationary Equilibrium for Different Levels of Trend Inflation

Inflation	1%	2%	10%	20%
Output in DM	0.41	0.40	0.35	0.30
Price of money	1.40	1.26	0.79	0.60
Avg. DM price	2.18	2.41	3.71	4.63
Stdev in DM price	0.11	0.13	0.24	0.30

drops close to 15 percent when going from an economy with money growth of 2 percent to another where it is 10 percent. As a result, higher inflation rates shift the whole distribution of prices to the right while increasing its dispersion.

Figure D8: Changes in the Stationary Equilibrium for Different Levels of Trend Inflation



Notes: Variables in the stationary equilibrium for different levels of trend inflation, $\mu = \{0.01, 0.02, 0.10, 0.20\}$. All variables are normalized with respect to their values under the benchmark parametrization when $\mu = 0.02$ (in red).

E Data

E.1 Aggregate data

My calibration frequency is quarterly. I use data for the period 1984Q1 - 2007Q4 obtained from Fred - St. Louis Fed, and some additional calibration targets form other papers, as discussed in the paper.

- Original series
  - Nominal gross domestic product (GDP): Fred - St. Louis Fed (`GDP`).
  - 3-Month Treasury Bill: Secondary Market Rate: Fred - St. Louis Fed (`TB3MS`).
  - Consumer Price Index for All Urban Consumers: All Items: Fred - St. Louis Fed (`CPIAUCSL`).
  - Federal Debt: Total Public Debt: Fred - St. Louis Fed (`GFDEBTN`).
  - Currency Component of M1: Fred - St. Louis Fed (`CURRSL`).
  - Total Checkable Deposits: Fred - St. Louis Fed (`TCDSL`).
  - Savings Deposits - Total: Fred - St. Louis Fed (`SAVINGSL`).
  - Small Time Deposits - Total: Fred - St. Louis Fed (`STDSL`).
- Computed series:
  - Stock of money: Defined as in Alvarez, Atkeson and Edmond (2009). Sum of currency, checkable deposits, and time and savings deposits. This is `M_AAE = CURRSL+TCDSL+SAVINGSL+STDSL`.
  - Velocity of money: Ratio of nominal GDP to stock of money. This is `V = GDP/M_AAE`.

E.2 SCF data

The data for the Survey of Consumer Finance for 2007 is obtained from https://www.federalreserve.gov/econres/scf_2007.htm. For all calculations, I use the weights provided with the survey (`WGT`). I use the Summary Extract Dataset. In particular, I use the variables:

- Total financial assets (`FIN`).

- Total liquid assets (all types of transaction account, `LIQ`).